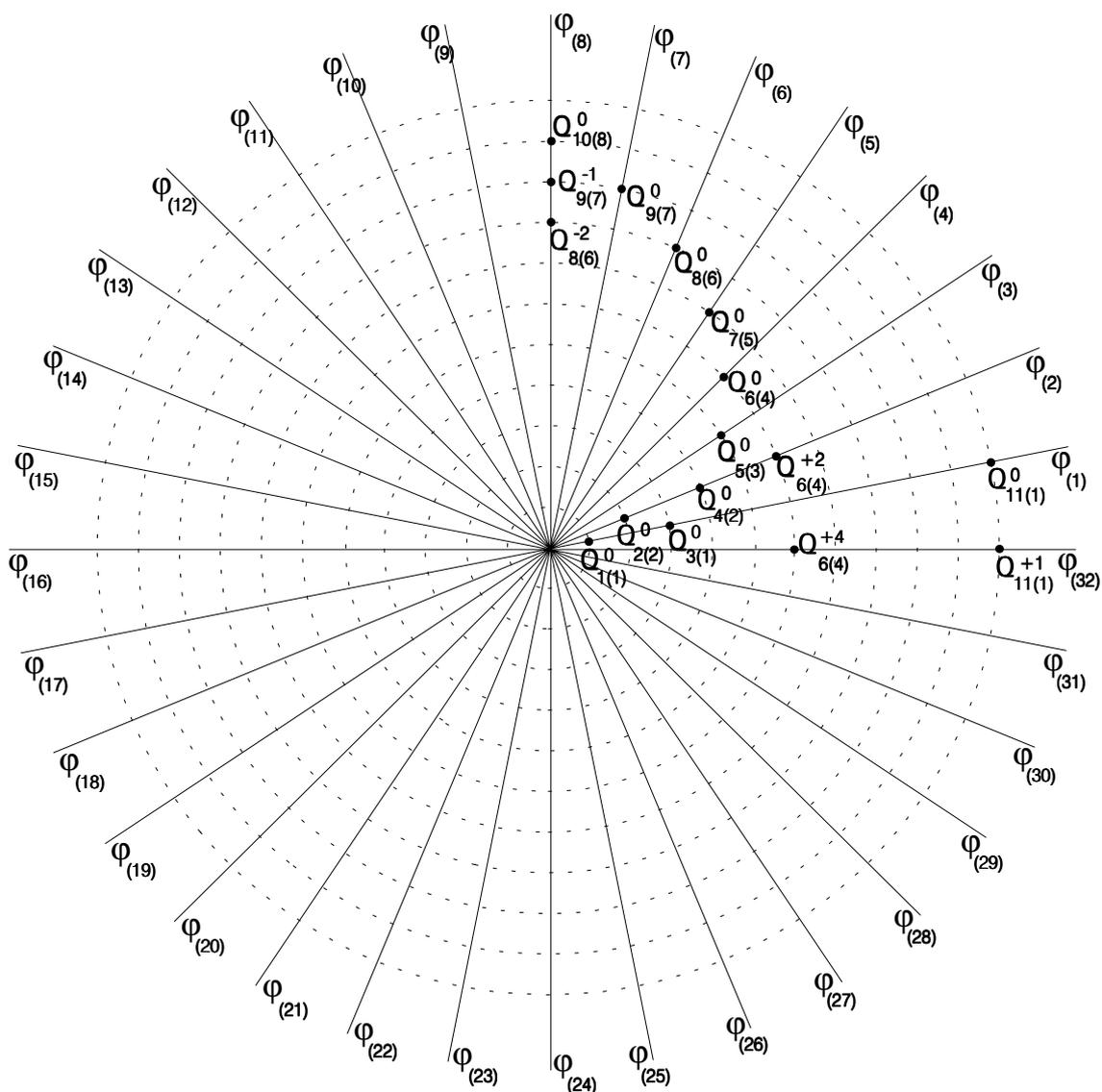


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# GEOMETRIZATION OF THE FUNDAMENTALS OF CHEMISTRY

Main point and purpose of the scientific knowledge and conception about existence of different evolutionary stages in sciences' development, presented by an example of chemistry fundamentals geometrization. Space mathematical models of different species of simple chemical objects and Mendeleev's periodical law.

(extended summary)



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## ANNOTATION

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In the monograph the problem of geometrization of the chemical language is formulated, grounded and partially solved. This monograph sets the beginning of a qualitatively new evolutionary stage in the chemistry development - *the stage of its geometrization*. The concept of **geometrization** of chemistry should be regarded in the most general sense of the term *geometry*. By this term we signify shortly the main trend in the development of each science during a given evolutionary stage. The **geometrization** can be qualified most generally as *a gradual transformation of the language of corresponding science into a system of such axiomatic formulations of different logically coherent and non-contradictory veracious statements of this science, which are characterized by a large-scale use of spatial mathematical concepts*. As examples for a quite advanced process of geometrization one can mention the mathematical formulations of the classic and quantum mechanics, electrodynamics, the theory of relativity, etc. In addition, the monograph presents an original view on the main point and purpose of the scientific knowledge and an original conception about existence of different evolutionary stages in sciences' development. All this makes the book interesting reading for a wide circle of readers (from students to highly qualified scientists) of different scientific subjects - chemists, mathematicians, physicists, mineralogists, philosophers etc.

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## ABSTRACTS OF THE CHAPTERS

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**Structure and peculiarities of the monograph:** The basic peculiarity of the monograph, its complicate conceptual structure, is presented. Thus the reader is provided by "a relief map of conceptual places" which he will visit during the reading. We comment briefly some additional peculiarities of the monograph related with the notions, ideas, and "instruments of production" used in the book.

**Introduction:** The main points of an original view on the essence and purpose of the scientific knowledge, which is both the philosophical frame and core of the monograph are presented. On the basis of this view one of the basic aims of the investigation, concerning the searching and revealing the connections between the scientific and philosophical-mystic knowledge is formulated and grounded.

**Chapter 1:** A system of three starting hypotheses, concerning the existence of different evolutionary stages in the development in scientific branches and sciences is formulated. Using these hypotheses the problem of chemistry geometrization is formulated in general form. The way for solving this problem is outlined by formulating a system of three basic problems to be solved during the next evolutionary stage of the chemistry development. The particular problems of the investigation are formulated and some of their peculiarities are commented. Basic logical and mathematical results, presented in monograph, are briefly reviewed.

**Chapter 2:** The general logical structure of the most developed scientific languages is analyzed to prepare the ground for a logical analysis and geometrization of the chemical language. It is shown that: (i) the general "instrument of production" of every science is its language; (ii) in fact, the analysis of the meaning content of a given science is an analysis of the logical structure the language of this science; (iii) the meaning content of the basic notion of each scientific language is defined by the system of relations between them, i.e. by the corresponding axiomatics.

**Chapter 3:** The foundations, construction, and analysis of an approximate spatial mathematical model  $\mathbf{P}=f(\mathbf{S},\mathbf{T}_0)$  of the set  $\mathbf{S}$  of different species of atoms and of simplified tabular short form  $\mathbf{T}_0$  of the

Periodic System are presented. A number of specific chemical-mathematical symbols are introduced, and the definitions of some basic chemical notions are revised. Defined are the Mendeleev's space  $V_M(3)$  and Mendeleev's coordinate system  $K_M=Oz\rho\phi$ , which are substantial in the geometrization of language of chemistry by providing the possibility to construct unique spatial mathematical models for both the species of simple chemical objects and of simple chemical processes. The three- and two-dimensional images are determined for a number of basic chemical notions (chemical element, group and period of Periodic System, maximal stoichiometric valence, etc.). A set of statements of various logical ranks (axioms, definitions, corollaries, properties, etc.) is formulated. A method for construction of spatial mathematical models of some basic chemical notions and relations is proposed and demonstrated with the example of the approximate model.

**Chapter 4:** The foundations, construction, and analysis of two precise spatial mathematical models  $Q=f_1(S,T_1)$  and  $G=f_1(S,T_2)$  of the set  $S$  of different species of atoms and of the corresponding long tabular forms  $T_1$  and  $T_2$  of the Periodic System are presented. In constructing and analyzing these two models: new chemical-mathematical symbols are introduced; a comparative analysis is carried out of the three spatial mathematical models,  $P$ ,  $Q$  and  $G$ ; the three- and two-dimensional images are determined for a number of basic chemical and physical notions (chemical element, period of Periodic System, occupied electron orbital, equal or similar electronic configurations of valence electron shell, etc.); a corresponding set of statements of various logical ranks (axioms, definitions, corollaries, properties, etc.) is formulated, analogous to that within the approximate model.

**Chapter 5:** The definitions of the basic chemical notions ionization, recombination and oxidation number are revised. The foundations, construction, and analysis of two generalized spatial mathematical model  $Q =F_1(S ,T_1)$  and  $G =F_1(S ,T_2)$  of the set  $S$  of different species of monoatomic ions are presented. In constructing and analyzing these two models: the list of specific chemical-mathematical symbols is expanded; the relations between some basic chemical notions are formulated mathematically by these symbols; a comparative analysis of both generalized models,  $Q$  and  $G$ , is carried out; three- and two-dimensional mathematical images are determined for monoatomic ions of equal oxidation number of different isotopes of one and a same chemical element; three- and two-dimensional mathematical images of different chemical elements are determined. For each of the two generalized models,  $Q$  and  $G$ , a set of statements of various logical ranks (axioms, corollaries, definitions, etc.) is formulated. This set generalizes the corresponding set of statements for the models  $Q$  and  $G$ .

**Chapter 6:** The logical sense and the ways for introducing the basic notions for denoting objects in scientific branches and sciences are analyzed. It is shown that: (i) from the logical point of view, all notions for denoting objects are mutually determined sets of properties which are defined by giving the corresponding systems of relations; (ii) during their development the natural sciences gradually deprive the language of any sensuous content and at the same time they transform it by axiomatic and structural methods; (iii) the real purpose of each natural science is to help us to feel the existing of the indivisible whole by the language.

**Chapter 7:** The qualitative meaning of the points and distances is separately analyzed for each of the developed models. In each of the point sets  $P$ ,  $Q$ ,  $G$ ,  $Q$  and  $G$  it is shown that: (i) the points are unique mathematical images of the corresponding mutually determined sets of properties; (ii) the distances between the points present simultaneously the differences between the corresponding mutually determined sets of properties and the reasons for these differences.

**Chapter 8:** It is shown that the fundamentals of chemistry can be logically divided into four relatively separated parts, in each of which a corresponding group of basic chemical notions is introduced.. The

central notions of the four parts and other basic notions are defined. The conceptual schemes, used for geometrization of each part of the fundamentals of chemistry, are formulated.

**Epilogue:** Contain the full text of the referee's report of Prof. Dr. Danail Bonchev and Prof. Dr. Ovanes Mekenyan, where the main results, presented in the monograph, are summarized. As a response of the referee's remarks, the main statement of the monograph - the statement about the impossibility of principle to describe the indivisible whole is further clarified.

**Appendix 1:** The problem of investigation of the regularities ruling the evolution of the scientific cognition is formulated and grounded in general form. Some basic categories and notions (taxon, species, genus, family, ontogenesis, phylogenesis etc.) are defined for the case of rational objects. Two generalizations of the biogenetical law are formulated for the case of rational objects. The concept of existence of qualitatively different evolutionary stages in the development of the separate branches of sciences, sciences and system of sciences is presented.

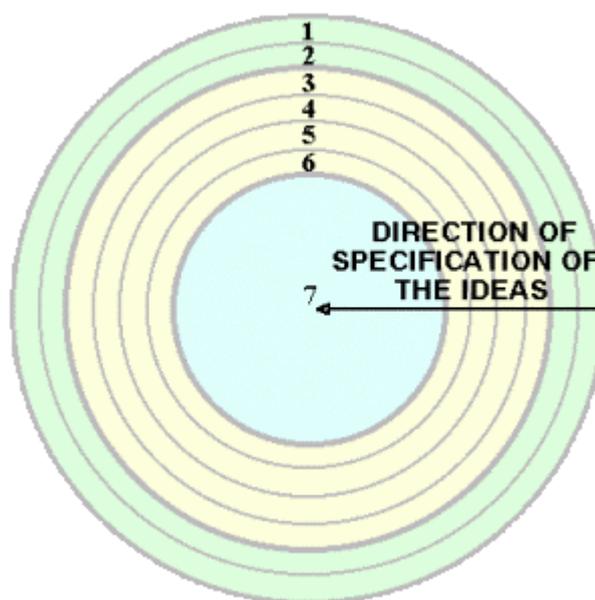
**Appendix 2:** A conceptual program for further investigations related as to the problem of geometrization of Chemistry, as well as to the far more general problems concerning the essence and purpose of the scientific knowledge and the different evolutionary stages through which it passes is presented.

**Appendix 3:** The correspondence between the symbols of the chemical elements, their two-dimensional mathematical images in models **P**, **Q** and **G**, introduced in the monograph, and the coordinates of these images in the plane  $z=0$  are presented in tabular form.

## EXTENDED SUMMARY

### STRUCTURE AND PECULIARITIES OF THE MONOGRAPH

1. View on the scientific knowledge main point and purpose (Introduction, Ch. 6, parts of Appendices 1 and 2);
2. Concept of qualitatively different kinds of evolutionary stages existence in the development of scientific languages (parts of Ch. 1, App. 1);
3. Formulation of the chemistry geometrization problem and outlining the way of its solving (part of Ch. 1);
4. Analysis of the scientific languages' structure (Ch. 2, part of Ch. 6);
5. Review of the general logical structure of chemistry fundamentals (Ch. 8);
6. Formulation of conceptual schemes, serving the chemistry fundamentals particular parts geometrization (parts of Ch. 3 and 8);
7. Geometrization of part of the chemistry fundamentals (Ch. 3-5, 7, App. 3, and 14 Figures).



This fundamental and interdisciplinary monograph is a wide and complicated multilayer structure of ideas, which have not been published yet. Three basic conceptual strata of rigid hierarchical subordination can be distinguished in this structure. In addition, the ideas of each stratum are connected each other by a net of logical relations.

The general conceptional stratum is composed of two relatively distinguished layers of ideas presenting the philosophical framework of the monograph. That is why, they frame the basic text (i.e. they are located out of it). The ideas of the first layer are formulated in the Introduction and are developed further in Ch. 6 (parts of them are sketched in Appendix 1 and at the end of Appendix 2). Similarly, a part of ideas belonging to the second layer is presented in the introduction of Ch. 1, while their extended presentation can be found in Appendix 1.

The first layer of the philosophical frame is a complete outlook on the scientific knowledge main point and purpose. This outlook is a result of a logical analysis of the western type of science principles. Being close to the outlook of the philosophical-mystic cognizant tradition, the outlook considered is still quite different from the western type of science point of view.

The second layer of the philosophical frame represents a complete conception about existence of qualitatively different evolutionary stages in the development of particular branches of science, sciences, and complexes of sciences. This conception is a result of the western type of science logical analysis as well.

The above described relatively distinguished layers of general ideas are both strongly connected to each other. Considered as an unity, they present a complete attempt to reveal the rules governing the scientific knowledge development as well as the final aim of this development.

Apparently, general ideas like the above described have their hypothetical nature. That is why, the general tasks of the monograph are:

**I.** To verify the outlook on the scientific knowledge main point and purpose and the concept of existence of qualitatively different evolutionary stages of its development. This will be done by the formulation, the strict grounding, and the partial solving of the chemistry language geometrization problem;

**II.** To reveal, at least partially, the connections between scientific and philosophical-mystic knowledge through a logical analysis of the basis of the western type of science, (the chemistry fundamentals in this particular case).

Those two general tasks are formulated in the Introduction. The main part of the monograph (Ch. 1-5 and 7-8, all Figures and App. 3) is dedicated to the solving of the chemistry language geometrization problem (this is mentioned in the title of the monograph, as well). The solution of the second general task is presented mainly in the Introduction and Ch. 2, 6.

Formulation of the philosophical frame has enabled the more specific problem for the chemistry geometrization to be formulated in general form and to be validated strictly enough, as well. In the author's opinion, even the partial solution of this problem, presented in the monograph, is a sufficient verification of both, the concept of qualitatively different stages in the scientific knowledge evolution existence and the outlook of the knowledge's main point and purpose.

Therefore, the second conceptional stratum of the monograph is a consecutive specification of the most general philosophical-evolutionary stratum ideas in the case of the chemistry language. It includes:

**1)** formulation of the chemistry geometrization problem and outlining the way of its solving (Ch. 1);

**2)** analysis of the logical structure of the scientific languages (Ch. 2 and a part of Ch. 6);

**3)** review of the chemistry fundamentals general logical structure (Ch. 8);

**4)** formulation of the conceptional schemes, used for the relatively separated parts of the chemistry fundamentals geometrization (parts of Ch. 3 and 8).

The formulated in Ch. 1 system of three initial hypotheses is the linkage between the philosophical-evolutionary conceptional stratum and the second more specific conceptional stratum. On their turn, the conceptional schemes, used for the relatively separated parts of the chemistry fundamentals geometrization, play the role of the linkages between the second conceptional stratum and the third most specific conceptional stratum.

The third stratum of the monograph is the most specific one and at the same time is the largest one (Ch. 3-5, parts of Ch. 6, 7, App. 3, and all Figures belonging to it). In fact, this stratum includes the real geometrization of part of the chemistry fundamentals. It includes:

- 1) logical analysis of the system of basic chemical notions;
- 2) constructing and analyzing of several different unique space mathematical models of a number of basic chemical notions.

Thus, we briefly sketched the conceptional structure of the monograph. Now it is necessary to describe the “tools” which are used.

## METHODS AND INSTRUMENTS

Logic is the main instrument used in all conceptional strata of the monograph. However, dialectical (i.e. “multidimensional”, “spatial”) logic is used in most general strata, instead of ordinary binary logic. At the same time, “dimensionality” of the used logic decreases, reaching “one-dimensional” (i.e. binary) logic, with specifying of the problems discussed. In addition to the logical methods and instruments, such powerful mathematical methods and instruments as coordinate geometrical method, set theory, mathematical modeling etc., are very efficiently used in the monograph.

Second peculiarity of the monograph (besides its complicated logical structure) is that in each of the three conceptional strata corresponding specific tools (ideas and concepts) are applied. Third peculiarity is the rather high concentration of relatively new and not common to most of the readers ideas.

## REVIEW OF THE MAIN IDEAS IN THE FIRST TWO STRATA

The normal way to review the major ideas is to start with the most general ideas in the monograph, i.e. with the outlook on the scientific knowledge main point and purpose. From a logical point of view, the main points in this outlook are just an explicit formulation of the two fundamental assumptions, lying at the basement of each science or a branch of science and the logical conclusions one can extract from them. These two assumptions are:

- 1) **The whole is that what really exists.** (This assumption is discussed in Ch. 6.)
- 2) **The whole is absolutely indivisible.** This means that it is not a sum of parts at all. (This assumption also is discussed in Ch. 6 and is illustrated by an example in the Introduction.)

The following conclusions directly result from these fundamental assumptions:

3) **The whole exists outside and independently of the language, while the language does not exist (and can not exist) outside and independently of the whole.** In other words, **the whole is beyond the language, i.e. it precedes all notions and ideas. Due to this the whole is neither notion, nor idea at all.**

4) **The whole can be pointed out and felt but, generally, can not be described by any means.**

5) **Sciences and branches of sciences do not describe but, pointed out the indivisible whole.**

6) **Each science or a branch of science points out the indivisible whole by the language through a corresponding system of imaginary parts.** (On one hand this system is visible manifestation of the indivisible whole in the language of the corresponding science or a branch of science. On the other, it introduces (defines) the set of basic notions of the corresponding science or a branch of science, i.e. the set of the corresponding imaginary parts, through which the science or the branch of science “describes” the indivisible whole.)

7) **The real aim and purpose of the scientific cognition is to help us to realize by the language the existence of the indivisible whole.**

8) **All natural sciences gradually deprive the language of any sensuous content and at the same time they transform it by axiomatic and structural methods, in order to point out the existence of the indivisible reality by the language.**

It is necessary to note expressly that the presented outlook on the main point and purpose of the scientific knowledge is formulated on the basis of logical analysis of the western type science (Ch. 2 and 6 are dedicated to this analysis). At the same time, this outlook is quite different from the usual to the western science one (main reasons for this difference are discussed in the Introduction) but, generally, is very close to the outlook of the philosophical-mystic cognitive tradition, and in particular to the east philosophical-mystic one (this similarity is illustrated by quotations of Shiva, Buddha, Shankara etc.). That is why, the problem of relations revealing between the scientific and philosophical-mystic knowledge is formulated and grounded in the Introduction as an extremely important for the contemporary science problem. Besides, it is pointed out that in order to solve this problem, the western science language should be transformed, in such a way to shift the emphasis from the analysis (i.e. from the parts, from the fragmentariness) to the synthesis (i.e. to the whole). A hypothesis has also been formulated, that the tendency for geometrization of the different sciences and branches of sciences is exactly this kind of evolutionary transformation of the western type of science language. In this way, although rather generally, the problem of the chemistry language geometrization is grounded.

The presented outlook on the scientific knowledge main point and purpose reveals the reasons for the evolution of the western science language and the direction of this evolution, as well. In other words, starting from this outlook in a natural to the logical approach way, we come to the formulation of:

- the concept of existence of qualitatively different evolutionary stages in the western science development;
- in general, the problem of geometrization of the scientific languages and in particular, the problem of chemistry language geometrization.

Briefly, we consecutively come to the second layer of the monograph philosophical frame and the next more specific conceptional stratum. The main points of this logical transition are presented in the introduction of Ch. 1 by the following system of three starting epistemological hypotheses:

**FIRST STARTING HYPOTHESIS:** The scientific knowledge evolution obeys rigorous laws. It is possible to reveal them by analyzing the qualitatively different evolutionary stages of the most developed sciences. The knowledge on these laws allows: **(1)** to determine the main trends according to which sciences from the lower evolutionary stages have to develop; **(2)** to formulate the *main evolutionary problem*, that has to be solved in each less developed science during the forthcoming stage of its evolution. (By formulating this main evolutionary problem and solving it step-by-step the development of such a science enters the next qualitatively new evolutionary stage.)

**SECOND STARTING HYPOTHESIS:** The analysis of the most developed sciences history indicates that our knowledge of various groups of phenomena has developed (and keeps developing) towards the creation of axiomatic formal theories describing their subjects (i.e. the corresponding groups of phenomena) in terms of spatial mathematical models. This trend of development reveals one of the principle laws ruling, in general, the evolution of the scientific knowledge.

**THIRD STARTING HYPOTHESIS:** The *main evolutionary problem* for the contemporary chemistry is the problem of its geometrization. By formulating and solving this problem the chemistry will enter in a qualitatively new stage of development, called the *stage of its geometrization*.

By the term *geometrization* we have tried to signify shortly and to point out the main trend in the development of each science during a given strictly specified evolutionary stage. It is clear (from the second starting hypothesis) that *the geometrization can be qualified most generally as a gradual transformation of the language of corresponding science into a such system of axiomatic formulations of different systems of logically consentient non-contradictoried veracious statements of this science, which are characterized by a large-scale exploitation of spatial mathematical concepts*.

Therefore, the problem of chemistry geometrization was formulated and grounded in general form by formulating the system of three starting hypotheses. Thus, we entered into the second conceptional stratum of the monograph. The further specifying of the ideas of this stratum includes:

- outlining the way for the geometrization of chemistry;

- review of the general logical structure of the chemistry fundamentals;
- formulation of the conceptional schemes used for the geometrization of chemistry fundamentals particular parts.

Below we summarize more important points of these conceptional layers.

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## OUTLINING THE WAY OF THE CHEMISTRY GEOMETRIZATION

In order to entirely geometrize chemistry one should solve the following set of three closely related principle problems:

1. To reveal the logical structure of the language of chemistry in order to formulate correctly the chemical notions and relations;
2. To construct entirely mathematical spatial representations of these notions and relations;
3. To formulate systems of logically consentient, non-contradictory and veracious chemical statements playing the role of axiomatics in further axiomatic formulations of the language of chemistry;
4. To prove logically the truth of those chemical statements which are not axioms, i.e. these statements to be deduced from the formulated chemical axiomes.

The complete solution of the above mentioned three problems is a large and profound task. So, in the study performed we confined ourselves to find out partial solutions for the first two problems, sufficient to find strictly and to demonstrate exactly, that the problem of the chemistry geometrization is solvable and, thus, to confirm the accuracy of the starting hypotheses. Briefly, we have confined to the geometrization of a part of the chemistry fundamentals.

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## LOGICAL STRUCTURE OF CHEMISTRY FUNDAMENTALS

A preliminary review of the most general logical structure of the chemistry language is necessary in order to answer the question “Which part of the chemistry fundamentals is geometrized really in the monograph?”. That is why, we shall at first define some basic notions necessary for the presentation of the chemistry language logical structure:

**DEFINITION I.** By the term *simple chemical object* we denote *each chemical object composed of a unique atomic nucleus and a corresponding electronic shell.*

**DEFINITION II.** By the term *complex chemical object* we denote *each chemical object composed of several (more than one) simple chemical objects.*

**DEFINITION III.** By the term *atom* we denote *each electrically neutral simple chemical object*, and by the term *ionized atom* (or *monatomic ion*) – *each electrically charged simple chemical object.*

From the above definitions is evident that the set of simple chemical objects includes all *species*<sup>1</sup> of *atoms* and *ionized atoms*, only. On its term, the set of complex chemical objects includes all *species* of *polyatomic ions*, *simple* and *complex molecules*, etc.

As notions, *simple* and *complex chemical objects* are closely related with the notions *structure*, *compositon*, and *construction*. The definitions of last three notions are:

**DEFINITION A.** By the term *structure* of a given complex object in an exactly specified energy state we denote this *unity* of a particular *compositon* and a particular *construction* which characterizes *uniquely the object in this energy state.*

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<sup>1</sup> Using *species* in this study we mean *the major subdivision of a genus or subgenus, regarded as the basic category of chemical classification*. We believe, that more particular meaning of *species* (instead of *kind*, for example) is closer to the essence of the book. This meaning accords with the following meanings of *species* (Random House Webster's): 1) Logic. a) one of the classes of things included with other classes in a genus. b) the set of things within one of these classes. 2) the major subdivision of a genus or subgenus, regarded as the basic category of biological classification, composed of related individuals that resemble one another, are able to breed among themselves, but are not able to breed with members of another species.

**DEFINITION B.** By the term *compositon* of a given complex object in an exactly specified energy state we denote *this set of relatively independent particular parts which characterizes uniquely the object in this energy state.*

**DEFINITION C.** By the term *construction* of a given complex object in an exactly specified energy state we denote *this particular mutual disposition of relatively independent parts, composing the object in the usual three-dimensional physical space, which characterizes uniquely the object in this energy state.*

In the last three definitions the meaning of *energy state* of a particular object is the value of internal energy of this object considered as a thermodynamic system (TDS). In thermodynamics the internal energy of TDS is defined as a difference between the total energy of this system and its external energy. Correspondingly, the external energy of TDS includes the energy of motion of the system as a whole and its potential energy in the field of external forces.

One can see from the above definitions that simple chemical objects have physical structure but, do not have chemical structure. Consequently, these objects are complex from a physical point of view, but are simple from the chemical point of view (that is why we have denoted them as simple chemical objects). In contrast, the complex chemical objects have both physical and chemical structure and these objects are complex from both physical and chemical point of view (that is why we have denoted them as complex chemical objects).

The above definitions allow us to outline the general logical structure of the chemistry fundamentals. We can present this structure by the following scheme:

**Table 1. General logical structure of the fundamentals of chemistry**

<p><b>I. CHEMISTRY OF SIMPLE OBJECTS</b> (chemistry of different species of atoms and different species of monatomic ions)</p>	
<p><b>1. STATICS</b> <b>of simple chemical objects</b> This parts includes the notions necessary for describing different species of simple chemical objects.</p>	<p><b>2. DYNAMICS</b> <b>of simple chemical objects</b> This parts includes the notions necessary for describing different processes of transformations of the species of simple chemical objects.</p>
<p><b>II. CHEMISTRY OF COMPLEX OBJECTS</b> (chemistry of different species of monatomic ions, simple and complex molecules etc.)</p>	
<p><b>3. STATICS</b> <b>of complex chemical objects</b> This parts includes the notions necessary for describing different species of complex chemical objects.</p>	<p><b>4. DYNAMICS</b> <b>of complex chemical objects</b> This parts includes the notions necessary for describing different processes of formation, destruction and transformations of the species of complex chemical objects.</p>

From a logical point of view this scheme shows that:

- A.** The chemistry fundamentals includes four relatively separated parts.
- B.** Each one of these four parts introduces a corresponding group of basic chemical notions.
- C.** Each notion group is centered on an exactly defined notion. Further in the text we shall refer this notion as a *central notion* of the corresponding notion group.

**D.** There are complex relationships of horizontal and vertical hierarchical subordination between the four separated parts of the chemistry fundamentals. To convince oneself in that we should note that both the dynamics of simple chemical objects and the statics of the complex ones can be developed if the statics of simple chemical objects has been previously developed. On its turn, the dynamics of the complex chemical objects cannot be developed unless the other three parts of the basis of chemistry have not previously been developed.

Let us now see which are the central notions in the four parts of the chemistry fundamentals and how they can be defined:

**1.** The *central notion* of the first part of the chemistry fundamentals is the notion *species of simple chemical objects*. We denote by this notion *the set of all simple chemical objects of the same physical composition (and of the same physical construction when they are in equivalent energy states)*. In this definition *physical composition* means *the corresponding uniquely defined (quantitatively and qualitatively) set of neutrons, protons and electrons composing a given simple chemical object*. Correspondingly, by *physical construction* we denote *the wave function of all physical particles composing a given simple chemical object*.

**2.** The *central notion* for the second part of the chemistry fundamentals is *simple chemical process*. We denote by this notion *each process which transforms the species of simple chemical objects*. Examples for such processes are: all processes of ionization and recombination of simple chemical objects; all processes of nuclear decay of simple chemical objects, etc.

**3.** The *central notion* for the third part of the chemistry fundamentals is *species of complex chemical objects*. In contrast to a simple chemical object, *each complex chemical object is a unity of chemical composition and chemical construction*. The notion *chemical composition* means *the set of different species of simple chemical objects, composing a particular complex chemical object*. The notion *chemical construction* means *the mutual disposition of the simple chemical objects, composing a particular complex chemical object in the three-dimensional physical space*. The notion *species of complex chemical objects* means *the set of all complex chemical objects of equal chemical composition and equal chemical construction when they are in equal energy states*.

**4.** The *central notion* for the fourth part of the chemistry fundamentals is *complex chemical process*. We denote by this notion *each process of: (i) formation of complex chemical objects; (ii) destruction of complex chemical objects; (iii) transformation of the species of complex chemical objects*.

Two basic questions need to be answered before formulation of the conceptional scheme of the first part of chemistry fundamentals geometrization (*statics* of different species of simple chemical objects): **1)** *What should be introduced in chemistry first of all?* **2)** *What is the exact way to its introduction?* To answer these questions we need to continue the logical analysis of the chemistry fundamentals.

From logical point of view chemistry is based on the following system of fundamental assumptions for existence:

**ASSUMPTION 1:** There exists a set **S** of different species of atoms.

**ASSUMPTION 2:** There exist sets of different processes, transforming the *species* of simple chemical objects. (Examples for such sets are the set of processes of ionization, the set of processes of recombination, the set of processes of  $\alpha$ -decay, etc.)

**ASSUMPTION 3:** There exist sets of different species of complex chemical objects. (Examples for such sets are the set of different species of polyatomic ions, the set of different species of simple molecules, the set of different species of complex molecules.)

**ASSUMPTION 4:** There exist sets of different processes of formation of complex chemical objects, of destruction of complex chemical objects or of transformation of the species of complex chemical objects. (Examples for such sets are the set of processes of formation of polyatomic ions, the set of processes of destruction of simple molecules, etc.)

From a logical point of view, each of these fundamental assumptions for existence is a *central axiom* of the corresponding part of the chemistry fundamentals, because it introduces the *central*

*notion* of this part and other basic notions, related to this central notion. Besides, the four fundamental assumptions for existence are mutually related, i.e. they form a *system of central axioms*. To support these we shall note that from the assumption for existence of the set of different species of atoms (i.e. from Assumption 1) and from the assumption for existence of the set of different processes of ionization (the latter is a part of Assumption 2) follows directly the existence of the set of different species of ionized atoms.

It is evident from the aforesaid that *the set S of different species of atoms is introduced in chemistry first of all*. This is the answer of the first question formulated above.

According to Assumption 1 and 2, there exist different species of simple chemical objects. Furthermore, according to Definitions I-III and A-C, the simple chemical objects are structureless from a chemical point of view. That is why, they cannot be differentiated in species by their *chemical structure*. The only way to determine the *species* of such objects is by defining a *system of relations* between their *sets of properties*. Therefore, the only chemical way to define the set **S** is by defining the *system of relations between sets of properties* of different elements of **S**. This is exactly the way that is used in chemistry. Indeed, all the forms of Periodic System (PS) are different particular representations of the system of relations between the sets of properties of different species of atoms. Therefore, from the logical point of view, the fifth fundamental assumption in the language of chemistry is:

**ASSUMPTION 5.** The set **S** is introduced in contemporary chemistry by different *implicit three-dimensional forms* of the Periodic System<sup>2</sup>.

Therefore, *the geometrization of the language of chemistry should be started by constructing mathematical representations of the three-dimensional forms of Periodic System (i.e. of the Mendeleev's Periodic law) which introduce the set S in chemistry*. This is the answer of the second question formulated above.

## CONCEPTIONAL SCHEMES AND MATHEMATICAL MODELS

The last layer from the second conceptional stratum includes conceptional schemes, used for the geometrization of four basic parts of the chemistry language. However, the formulation of these schemes is closely related to the formulation of the basic ideas of the monograph third stratum, including the actual geometrization of the statics and a part of the dynamics of simple chemical objects. That is why, conceptional schemes and basic ideas of the third stratum will be presented simultaneously. Some abbreviations will be used in this presentation: **PS** - Periodic System; **SFPS** - Short Form of the Periodic System; **LFPS** - Long Form of the Periodic System. In addition, we will use following symbols:

- Capital **S** denotes *the set of all species of atoms*, while small **s** denotes *a particular mathematical element of this set* (i.e. a particular species of atoms);
- Capital **SO** denotes *the set of all species of simple chemical objects*, while small **so** denotes *a particular mathematical element of this set* (i.e. a particular species of simple chemical objects);
- Capital **C** denotes *the set of all species of ionized atoms*, while small **c** denotes *a particular mathematical element of this set* (i.e. a particular species of ionized atom);
- $N_n$  denotes *the number of neutrons in a particular species of simple chemical objects*;
- $N_p$  denotes *the number of protons in a particular species of simple chemical objects*;
- $N_e$  denotes *the number of electrons in a particular species of simple chemical objects*.

<sup>2</sup> We state that the *three-dimensional forms* of the Periodic System are defined *implicitly* because the numbers of protons and electrons of different species of atoms (i.e. of different isotops) of a same chemical element are presented by two-dimensional forms of Periodic System, while the number of neutrons is presented in a separated table.

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## I. GEOMETRIZATION OF THE SIMPLE CHEMICAL OBJECTS STATICS

The first step of the geometrization of the simple chemical objects statics is *a construction of unique spatial mathematical models of the set  $\mathbf{S}$  which, at the same time, is a veracious mathematical models of PS*. However, it is known that different forms of PS present the Mendeleev's law with differing degrees of accuracy and completeness. So, three different space mathematical models (one approximate and two more precise) of set  $\mathbf{S}$  of different species of atoms and Mendeleev's law have been grounded, constructed and analyzed in Ch. 3 and 4 of the monograph. Each of these models is based on a particular tabular form of PS, denoted as *starting* (or *originating*) *table* for the corresponding mathematical model. Common features of those three starting tables used is that in each one of them the symbols of different species of atoms occupy identical positions in the corresponding cells. On their turn, common features of those three mathematical models originated from the three starting tables are that they are constructed as follows:

- in one and a same space  $V_M(3)$ ;
- with respect to one and a same coordinate system  $K_M$ ;
- by one and a same method (for the first time this method is demonstrated in Ch. 3 as an example of the approximate mathematical model and later in Ch. 4 as two more precise mathematical models).

The main difference between the three models is that they originate from three different forms of PS. The approximate mathematical model originates from a *simplified* tabular Short Form of the Periodic System (i.e. *simplified* SFPS), named  $T_0$  and shown in [Fig. 1](#). The two more precise mathematical models originate from two different 32-column tabular LFPS, named respectively  $T_1$  and  $T_2$  and shown in [Fig. 2.1](#) and [Fig. 2.2](#).

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### 1. APPROXIMATE SPATIAL MATHEMATICAL MODEL OF DIFFERENT SPECIES OF ATOMS

The conceptional scheme used in constructing the approximate mathematical model of the simplified tabular form  $T_0$  (by which the set  $\mathbf{S}$  of different species of atoms can be introduced in chemistry), consists of following four steps:

- 1) Construction of a space in which the *species* of different simple chemical objects (atoms and monatomic ions) can be represented uniquely;
- 2) Choice of coordinate system in this space which allows to represent uniquely the *species* of simple chemical objects;
- 3) Construction of the desired unique spatial mathematical model of the set  $\mathbf{S}$  which presents the corresponding starting form of PS as well. This step consists in the choice of a set of mathematical objects to represent uniquely the *species* of atoms in a previously chosen coordinate system in a previously constructed space. The set of these mathematical objects-images should be picked so that the system of relations between them to be a veracious mathematical model of the corresponding tabular form of PS;
- 4) Demonstration of the uniqueness and the veraciousness of the constructed spatial mathematical model. In this step one proves that:
  - the constructed mathematical objects really present uniquely the *species* of atoms with respect to the previously chosen coordinate system in the constructed space;
  - the relationships between the mathematical images of different species of atoms are really a veracious mathematical model of the regularities presented in the corresponding tabular form of PS.

The first two steps in this conceptional scheme are very important because they introduce the space and the coordinate system used later to construct the remaining mathematical models of the chemistry

fundamentals (incl. those used in constructing the two more precise mathematical models of  $\mathbf{S}$  and two mathematical models of  $\mathbf{S}$ ). That is why, we define below this space and choose a coordinate system therein.

Let  $V_M(3)$  be such a three-dimensional metric Euclidean space onto the field of the real numbers, in which each of the three dimensions has a different qualitative meaning. Furthermore, let  $K_M = \text{Oz}\rho\varphi$  be a specially chosen cylindrical coordinate system in  $V_M(3)$ , such that:

**A)** The three coordinates have the following different qualitative meaning:

- z-coordinates presents the number of neutrons in the simple chemical objects;
- $\rho$ -coordinates presents the number of protons in the simple chemical objects;
- $\varphi$ -coordinates presents the periodicity in the change of the chemical properties of different species of atoms. (This complex periodicity is presented with a different degrees of accuracy and completeness in the different starting forms of PS.)

**B)** The origin  $\mathbf{O}$  of the coordinate system  $K_M$  coincide with the natural zero point of  $V_M(3)$ . (It is clear from the qualitative meaning of the three dimensions of  $K_M$  that: **(i)**  $V_M(3)$  has a natural zero point; **(ii)** The qualitative meaning of this point is a lack of protons, neutrons, and chemically active electrons.)

The above defined space  $V_M(3)$  and the coordinate system  $K_M$  are named Mendeleev's space and Mendeleev's coordinate system (the subscript index  $M$  in their symbols presents shortly these names). They are substantial in the chemistry geometrization by making it possible to construct unique spatial mathematical images of both the *species simple chemical objects* and of *simple chemical processes* (i.e. the processes transforming the *species of simple chemical objects*).

By defining  $V_M(3)$  and  $K_M$  we made the first two steps towards the proof of the following theorem:

**THEOREM 1.** There exists such a unique spatial mathematical model of different species of atoms which, at the same time, is a veracious mathematical model of the simplified tabular SFPS.

This theorem plays the role of a basic statement for the approximate model. The next, third, step in proving the theorem is the actual construction of the model with respect to  $K_M$  in  $V_M(3)$ . For this purpose we have defined a invertible map  $f$  from  $\mathbf{S}$  onto the corresponding set of points  $\mathbf{P} = \mathbf{f}(\mathbf{S}, \mathbf{T}_0)$  in the Mendeleev's space. This invertible map is defined by the following lemma:

**LEMMA 1.** Each species of atoms can be uniquely presented in the space  $V_M(3)$  by a corresponding point  $p$  with cylindrical coordinates  $(z, \rho, \varphi)$  in  $K_M$ , defined according to the rules:

- A1.** z-coordinate is equal to the number of neutrons in the nucleus of the presented species of atoms;
- A2.**  $\rho$ -coordinate is equal to the number of protons in the nucleus of the presented species of atoms;
- A3.**  $\varphi = j(2\pi/8)$ , where  $j=1, \dots, 8$  is the number of the column (i.e. group) of the simplified SFPS, containing the presented species of atoms.

By formulating this lemma we constructed the approximate mathematical model. In the final, fourth, step of the proof of the theorem one must demonstrate that the set of point  $\mathbf{P} = \mathbf{f}(\mathbf{S}, \mathbf{T}_0)$  is really an unique mathematical model of  $\mathbf{S}$  and a veracious mathematical model of  $\mathbf{T}_0$ . For this purpose we have proved that:

1. The rules (A1, A2, A3) define really a invertible map (i.e. the map  $f$  is really a one-to-one correspondence);
2. The mathematical image of each chemical element consists of the mathematical images of all species of atoms (i.e. isotopes) of this chemical element only;
3. The mathematical image of each column of  $\mathbf{T}_0$  (i.e. the mathematical image of each group of the starting simplified SFPS) consists of the mathematical images of all chemical elements of this group only;
4. The order of chemical elements in each column (group) in the starting table  $\mathbf{T}_0$  is similar to that of their mathematical images in the mathematical image of this column;

5. The mathematical image of each period of  $T_0$  consists of the mathematical images of all chemical elements of this period only;

6. The order of chemical elements in each period of  $T_0$  is similar to that of their mathematical images in the mathematical image of this period;

7. The order of groups and periods in  $T_0$  is similar to that of their mathematical images.

During the processes of constructing and analyzing of the approximate model the following goals are achieved in Ch. 3:

- a number of specific chemical-mathematical symbols are introduced;
- the definitions of some basic chemical notions are revised;
- the three- and two-dimensional images are determined for number of basic chemical notions (chemical element, group and period of PS, maximal stoichiometric valence, etc.);
- a net of mathematical statements of various logical range (axioms, definitions, corollaries, properties, etc.) is formulated;

Briefly, by constructing and analyzing the approximate model:

- a main part of tools, necessary for the geometrization of chemistry, is introduced;
- the developed method for constructing spatial mathematical models of basic chemical notions is demonstrated.

## 2. TWO MORE PRECISE MATHEMATICAL MODELS OF DIFFERENT SPECIES OF ATOMS

Three starting tabular forms of PS ( $T_0$ ,  $T_1$  and  $T_2$ ) are analyzed in the beginning of Ch. 4. There is a number of important conclusions formulated about the logical structure of the both, tabular forms of PS and Mendeleev's law of periodicity, on the basis of this analysis. Two more precise spatial mathematical models of the set  $S$  and Mendeleev's Periodical law are constructed and analyzed by the method, demonstrated in Ch. 3. As a starting table for the first more precise model serves table  $T_1$  (Fig. 2.1), while for the second one serves table  $T_2$  (Fig. 2.2). The conceptional schemes used for constructing of both models include just the last two steps of the approximated model conceptional scheme (due to the fact that all models concerning different *species* of simple chemical objects are constructed with respect to  $K_M$  in the space  $V_M(3)$ ).

The basic statements of these two models are:

**THEOREM 2.1.** There exists such a unique spatial mathematical model of different species of atoms which, at the same time, is a veracious mathematical model of the starting table  $T_1$ .

**THEOREM 2.2.** There exists such a unique spatial mathematical model of different species of atoms which, at the same time, is a veracious mathematical model of the starting table  $T_2$ .

The first step in proving these theorems is to construct the desired models with respect to  $K_M$  in the space  $V_M(3)$ . For this purpose, an invertible map  $f_1$  from the set  $S$  onto the corresponding sets of points  $Q=f_1(S, T_1)$  and  $G=f_1(S, T_2)$  of  $V_M(3)$  is defined by the following lemma:

**LEMMA 2.** Each species of atoms can uniquely be presented in the space  $V_M(3)$  by a corresponding point with cylindrical coordinates  $(z, \rho, \varphi)$  with respect to  $K_M$ , defined according to the rules:

**B1.**  $z$ -coordinate is equal to the number of neutrons in the nucleus of the presented species of atoms;

**B2.**  $\rho$ -coordinate is equal to the number of protons in the nucleus of the presented species of atoms;

**B3.**  $\varphi = n(2\pi/32)$ , where  $n=1, \dots, 32$  is the number of this column in the corresponding starting table ( $T_1$  or  $T_2$ ), which contains the presented species of atoms.

By formulating this lemma the two more precise mathematical models are constructed. The second, final, step in proving of Theorems 2.1 and 2.2 contains the examination of the uniqueness and the veracity of these models. For this purpose it is proved that each of the both new mathematical

models,  $Q=f_1(S,T_1)$  and  $G=f_1(S,T_2)$ , obeys seven conditions which are analogous to the corresponding conditions for the approximate model.

During the processes of constructing and analyzing the two more precise models the following goals are achieved in Ch. 4:

- some new chemical-mathematical symbols are introduced;
- a comparative analysis of the three different spatial mathematical models,  $P=f(S,T_0)$ ,  $Q=f_1(S,T_1)$  and  $G=f_1(S,T_2)$  is carried out;
- the three- and two-dimensional images of a number of basic chemical and physical notions (chemical element, period of PS, filling up electron orbital, equal or similar electronic configuration of valence electron shell, etc.) are determined;
- a corresponding net of mathematical statements of various logical ranges (axioms, definitions, corollaries, properties, etc.) analogous to the net of mathematical statements in approximate model is formulated.

### 3. MATHEMATICAL MODELS OF DIFFERENT SPECIES OF MONATOMIC IONS

The geometrization of the part of chemistry language introducing the set  $S$  of different species of atoms, and chemical and physical notions related to it, is accomplished by constructing and analyzing of the two more precise models. However, the set  $S$  of different species of simple chemical objects includes the subset  $C$  of different species of ionized atoms (in addition to the subset  $S$  of different species of atoms). That is why, to accomplish the geometrization of statics of simple chemical objects it is necessary to construct spatial mathematical models of different *species of monatomic ions*. That is done in Ch. 5.

The notion *species of monatomic ions* is closely related to the notions *ionization*, *recombination* and *oxidation number* (or *oxidation state*) which belong to the second part of the chemistry fundamentals - dynamics of the simple chemical objects. That is why, the successful modelling of this notion requires correct definitions of the three related notions mentioned above. On the other hand, these notions are not correctly defined in the to-date chemistry. That is why, we shall revise their definitions as follows:

**DEFINITION IV.** By the term *ionization* of a given simple chemical object (i.e. atom or ionized atom) we denote each process in which the number  $N_e$  of electrons of this object is changed (without changing the numbers  $N_n$  and  $N_p$  of its neutrons and protons) in a such manner that the object becomes (or remains) electrically charged.

In other words, the ionization is a process of transformation of an atom of a particular species to some species of ionized atom, or of a particular species of ionized atom to another species of ionized atom.

**DEFINITION V.** By the term *recombination* of a given ionized atom (i.e. of an electrically charged simple chemical object) we denote each process of change of the number  $N_e$  of electrons of this object (without changing the numbers  $N_n$  and  $N_p$  of its neutrons and protons) such that the atom becomes electrically neutral.

In other words, the recombination is a process of transformation of some species of ionized atom to the corresponding species of atom.

**DEFINITION VI.** By the term *oxidation number* (or *oxidation state*) of a given monatomic ion we denote the number of electrons that should, either really or formally, be added or removed in order to transform this ion into atom (i.e. into an electrically neutral simple chemical object).

It is noticed from the last definition that the notion *monatomic ion* is a generalization of the notion *atom*, because each atom can be considered as an monatomic ion of oxidation number equal to zero.

Based on the above definitions, two different spatial mathematical models of the set  $S$  of different species of simple chemical objects are constructed and analyzed. Each of this models is a generalization of the corresponding more precise mathematical model ( $Q$  or  $G$ ) in the case of arbitrary

species of simple chemical objects - atoms and monatomic ions. As basic statements for the both generalized models could serve the following two theorems:

**THEOREM 3.1.** There exists such a unique spatial mathematical model of the set  $S$  of different species of simple chemical objects, which is a generalization of the model  $Q=f_1(S,T_1)$ .

**THEOREM 3.2.** There exists such a unique spatial mathematical model of the set  $S$  of different species of simple chemical objects, which is a generalization of the model  $G=f_1(S,T_2)$ .

The first step in proving these theorems includes the constructing of the two generalized mathematical models with respect to  $K_M$  in the space  $V_M(3)$ . For this purpose a invertible map  $F_1$  from the set  $S$  onto the corresponding sets of points  $Q = F_1(S, T_1)$  and  $G = F_1(S, T_2)$  of  $V_M(3)$  is defined by the following lemma:

**LEMMA 3.** Each species of simple chemical objects can uniquely be presented in  $V_M(3)$  by a corresponding point with cylindrical coordinates  $(z, \rho, \varphi)$  with respect to  $K_M$ , defined according to the rules:

- C1.**  $z$ -coordinate is equal to the number of neutrons in the nucleus of the presented species of simple chemical objects;
- C2.**  $\rho$ -coordinate is equal to the number of protons in the nucleus of the presented species of simple chemical objects;
- C3.**  $\varphi = (k - h)(2\pi/32)$ , where:  $k=1, \dots, 32$ , is the number of that column in the corresponding starting table ( $T_1$  or  $T_2$ ), which contains the presented species of simple chemical objects when they are electrically neutral; the integer number  $h$  is equal to the oxidation number of the presented species of simple chemical objects.

By formulating this lemma the two generalized mathematical models are constructed. The second step in proving both theorems shows that the corresponding sets of points,  $Q$  and  $G$ , are unique mathematical models of the set  $S$  of different species of simple chemical objects.

During the processes of constructing and analyzing of the two generalized models the following goals are achieved in Ch. 5:

- the list of specific chemical-mathematical symbols is expanded;
- the relations between some basic chemical notions are formulated mathematically by the symbols mentioned;
- a comparative analysis of the both generalized models,  $Q$  and  $G$ , is carried out;
- the three- and two-dimensional mathematical images are determined for monatomic ions of equal oxidation number of different isotopes of one and a same chemical element;
- the three- and two-dimensional mathematical images of different chemical elements are determined;
- for each of the two generalized models,  $Q$  and  $G$ , a net of mathematical statements of various logical ranges (axioms, consequences, definitions, etc.) is formulated. This net generalizes the corresponding net of mathematical statements for the models  $Q$  and  $G$ .

Especially important is the Corollary 23 of the two generalized models,  $Q$  and  $G$ , which states that the change in the oxidation number of a given species of simple chemical objects leads to a rotation of their mathematical images at an angle  $\Delta\varphi_{(n)}^{h,t}$  around axis  $Oz$  and this angle depends only on the difference  $(h - t)$  between the oxidation numbers of the simple chemical objects at the beginning and at the end of the corresponding process of ionization or recombination, i.e.:

$$\Delta\varphi_{(n)}^{h,t} = \varphi_{(n)}^h - \varphi_{(n)}^t = [2\pi(h - t)]/32$$

It is clear, from the mentioned corollary, that all processes of ionization or recombination of simple chemical objects can be mathematically presented in unique way in the space  $V_M(3)$  by corresponding rotation operators of their mathematical images around the axis  $Oz$ .

The investigation of the constructed spatial mathematical models have been accomplished in the Ch. 7 with an analysis of the qualitative meaning of the points and the distances between them in each one of the models. The results of this analysis are:

1) Each point of the sets  $\mathbf{P}$ ,  $\mathbf{Q}$ ,  $\mathbf{G}$ ,  $\mathbf{Q}$  and  $\mathbf{G}$  is an unique mathematical image of the corresponding species of simple chemical objects (i.e. of the corresponding relatively defined set of properties without any particular real bearer).

2) The distances between various points in each of these sets reveal:  
the differences between the corresponding relatively defined sets of properties;  
the physical and chemical reasons for these differences.

## CONCEPTIONAL SCHEMES FOR GEOMETRIZATION OF THE OTHER PARTS OF THE CHEMISTRY FUNDAMENTALS

In previously discussed chapters we presented conceptional schemes and some important points of the constructed spatial mathematical models of different species of simple chemical objects and Mendeleev's Periodical law. In other words, we have presented the main ideas of the third most concrete stratum of the monograph and a part of ideas belonging to the last layer of the second stratum. In the final chapter (Ch. 8) of the monograph the conceptional schemes for geometrization of the other three parts of the chemistry fundamentals are presented.

### II. DYNAMICS OF SIMPLE CHEMICAL OBJECTS

Presenting the general logical structure of the chemistry fundamentals we could summarize that:

- the notion *simple chemical process* is a central notion for the second part of the chemistry fundamentals;
- the *species* of simple chemical objects is uniquely determined by their *physical composition*.

From these statements it follows that each *simple chemical process* can uniquely be presented mathematically as a linear combination of three independent basic kinds of simple chemical processes. Each of these processes changes only the number of one of the three species of physical particles, constructing a given simple chemical object.

On the other hand, in each of the constructed mathematical models:

- $z$ -coordinates of mathematical images present the number of neutrons;
- $\rho$ -coordinates of mathematical images present the number of protons;
- $\varphi$ -coordinates of mathematical images implicitly present the number of electrons in the corresponding species of simple chemical objects.

What follows from the above analysis is:

1) The simple processes of the first basic kind change only the number of neutrons in the nuclei of simple chemical objects. That is why, these processes can uniquely be presented mathematically in  $V_M(3)$  by operators acting only on the  $z$ -coordinates of the corresponding mathematical images.

2) The simple processes of the second basic kind change only the number of protons in the nuclei of simple chemical objects. That is why, these processes can uniquely be presented mathematically in  $V_M(3)$  by operators acting only on the  $\rho$ -coordinates of the corresponding mathematical images.

3) The simple processes of the third basic kind change only the number of electrons in simple chemical objects. That is why, these processes can uniquely be presented mathematically in  $V_M(3)$  by operators acting only on the  $\Phi$ -coordinates of the corresponding mathematical images. (All simple chemical processes, changing the oxidation number, belong to this basic kind of processes.)

4) Each simple chemical process which transforms in arbitrary way the species of simple chemical objects can uniquely be represented mathematically as a linear combination of the three different kinds of operators listed above.

The last statement accomplished the conceptional scheme for geometrization of the chemistry fundamentals second part - dynamics of simple chemical objects.

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### III. STATICS OF COMPLEX CHEMICAL OBJECTS

The central notion of the third basic part of the language of chemistry is *species of complex chemical objects*. In order to construct unique spatial mathematical images of the various species of complex chemical objects one has to construct:

1) The Cartesian product of the space  $V_M(3)$  (in which the *species* of simple chemical objects are uniquely represented) and the usual three-dimensional physical space  $V_E(3)$  (in which the *mutual disposition* of the simple chemical objects is uniquely represented). Thereby we obtain a six-dimensional mathematical space  $V_S(6) = V_M(3) \times V_E(3)$  which is referred to as *space of the chemical structures*.

2) A special six-dimensional coordinate system  $K_S=Oz\rho\phi\psi$  (denoted as *coordinate system of the chemical structures*) in the space  $V_S(6)$ . This coordinate system is constructed as a Cartesian product of the Mendeleev's coordinate system  $K_M=Oz\rho\phi$  and any coordinate system  $K_E=O\psi\phi$  in the usual physical space  $V_E(3)$ , i.e.  $K_S = K_M \times K_E$ . It should be especially noted that the origin of the coordinate system  $K_S$  is a six-dimensional point  $O$  in the space of the chemical structures  $V_S(6)$ . The projection of this point onto the Mendeleev's space  $V_M(3)$  coincides with the natural zero of this space (i.e. with the origin of the Mendeleev's coordinate system  $K_M$ ).

It is easily noticed that all the *species* of complex chemical objects can uniquely be presented mathematically in the space  $V_S(6)$ . For this purpose, it is necessary to present all simple chemical objects, composing a given complex chemical object, by six-dimensional radius vectors (with respect to the six-dimensional coordinate system of chemical structures  $K_S$ ). The first three coordinates of each six-dimensional radius vector are defined with respect to  $K_M=Oz\rho\phi$  in  $V_M(3)$  and represent mathematically in a unique manner the *species* of the simple chemical objects constructing a given complex chemical object (i.e. they represent the *chemical composition* of the corresponding species of complex chemical objects). The last three coordinates of the radius vectors are defined with respect to  $K_E=O\psi\phi$  and uniquely represent mathematically in a unique manner the *mutual disposition* into the usual physical space  $V_E(3)$  of the corresponding simple chemical objects constructing a given species of complex chemical objects (i.e. they represent the *chemical construction* of this species of complex objects).

The later item accomplished the conceptional scheme for chemistry fundamentals third part geometrization - statics of complex chemical objects.

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### IV. DYNAMICS OF COMPLEX CHEMICAL OBJECTS

A *complex chemical process* is a central notion for the fourth part of the chemistry fundamentals. By this term we denote each process of: i) complex chemical objects formation; ii) complex chemical objects destruction; iii) complex chemical objects transformation.

Therefore, in contrast to simple chemical processes (transforming the *species* of simple chemical objects, only), the complex chemical processes can transform not only the *species* of the involved simple chemical objects but their *mutual location* in the usual physical space  $V_E(3)$  as well. The later statement outline the way of constructing of unique mathematical presentations for all the kinds of complex chemical processes, as follows:

1) Each transformation of the *species* of involved simple chemical objects can be uniquely presented mathematically by a corresponding linear combination of three basic kinds of operators acting onto the coordinates of the mathematical images of these objects in the Mendeleev's space  $V_M(3)$ .

2) Each transformation in the *mutual location* of involved simple chemical objects can be uniquely presented mathematically by a corresponding linear combination of operators of translation and rotation acting onto the points, presenting the location of these objects in the usual physical space  $V_E(3)$ .

Therefore, each complex chemical process can be uniquely presented mathematically in the space of chemical structures  $V_S(6)$  by the corresponding linear combination of operators acting onto the coordinates of the six-dimensional radius vectors which represent the simple chemical objects taking part in a particular complex chemical process.

The later statement accomplished the conceptional scheme for the chemistry fundamentals last fourth part geometrization - dynamics of complex chemical objects.

## CONCLUSION

The problem of chemistry language geometrization is solved on a conceptional level by a formulation of conceptional schemes for the chemistry fundamentals four parts geometrization. However, it should be exactly noted that the conceptional schemes are just the beginning of the process of these basic parts real geometrization. Further, this process continues by a logical analysis of the notions from each corresponding part, achieving their proper defining and with mathematical modeling of the already corrected notions as well. That is why, a reasonable question could be, which parts of the chemistry fundamentals is already geometrized in this monograph. The answer is that the first part of the chemistry fundamentals (statics of the simple chemical objects) is geometrized entirely by constructing and analyzing of spatial mathematical models of the species of simple chemical objects. In addition, the fundamentals of the second part geometrization (dynamics of the simple chemical objects) are laid.

## DESCRIPTION OF FIGURES

Ideas of the most concrete third strata of monograph are illustrated by 14 Figures and one Appendix. Three starting tables  $T_0$ ,  $T_1$ , and  $T_2$  are presented in [Fig. 1](#), [Fig. 2.1](#), and [Fig. 2.2](#). The cylindrical coordinates  $(z, \rho, \varphi)$  of an arbitrary chosen point  $p$  with respect to  $K_M$  are shown in [Fig. 3](#). [Fig. 4](#) presents the location of mathematical images of different isotopes of the chemical element with order number  $i=1$ . The eight semiplanes  $zO\varphi_{(j)}$  of mathematical images of different columns (groups) of the simplified SFPS are presented in [Fig. 5](#). [Fig. 6.1](#) is connected with the approximate mathematical model  $P=f(S, T_0)$  and shows two-dimensional mathematical images of the first 11 chemical elements in the plane  $z=0$  of  $V_M(3)$  and the auxiliary line  $\mathcal{P}$ , used for verification of the approximate model in the two-dimensional case. [Fig. 6.2](#) is similar to [Fig. 6.1](#) but the view point is outside the axis  $Oz$ . This figure guides the introduction of the auxiliary surface  $\mathcal{B}$ . [Fig. 7.1](#) illustrates the defining of the auxiliary surface  $\mathcal{B}$ , used for verification of the approximate model for the case of three-dimensions. [Fig. 7.2](#) presents for the first 11 chemical elements; auxiliary surface  $\mathcal{B}$ ; semiplanes  $zO\varphi_{(j)}$ ; auxiliary lines  $L_{i(j)}$  (containing three-dimensional mathematical images of chemical elements); two-dimensional mathematical images  $P_{i(j)}$  of chemical elements in the plane  $z=0$ . This Figure is used for verification of the approximate model in case of three-dimensions. [Fig. 8.1](#) and [Fig. 8.2](#) present two-dimensional mathematical images of the first 11 chemical elements in the plane  $z=0$ , for each of both more precise mathematical models  $Q=f_1(S, T_1)$  and  $G=f_1(S, T_2)$ . [Fig. 9.1](#) and [Fig. 9.2](#) present two-dimensional mathematical images of the first 11 chemical elements in the plane  $z=0$  and following species of monatomic ions  $C^{+2}$ ,  $C^{+4}$ ,  $O^{-2}$ ,  $F^{-1}$ ,  $Na^{+1}$  for each of the couple of generalized mathematical models  $Q = F_1(S, T_1)$  and  $G = F_1(S, T_2)$ .

A brief preliminary comment is necessary to clarify the meaning of Appendix 3. Up to now, the entire information for properties of given chemical element according to the chemistry language has been related to its symbol. In other words, the symbols of chemical elements serve as names (labels) of the corresponding “chemicals databases”. However, specific chemical-mathematical symbols are introduced in the mathematical models constructed to denote the mathematical images of chemical elements instead of their well known chemical symbols. So, a detailed Table of the relation between old and new names of “chemicals databases” is given in App. 3 for each of the constructed mathematical models. Coordinates of two-dimensional mathematical images of chemical elements for each of models are also given in the same Table. We hope that this Table will facilitate the understanding of the proposed mathematical models and their usage from chemists.

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### **THE REMAINING...**

Up to here we presented basic ideas of those parts of the monograph which are directly related to the grounding, formulation, and solution of the problem of the chemistry geometrization to some extent (i.e. related to the title of the book). The choice of this “guiding light” of this summary leaves untouched some of most important ideas of the monograph. These ideas are presented in Ch. 6, second part of the Epilogue, and the first two Appendices<sup>3</sup>, and are summarized in the motto of the monograph as well. That is why, I will finish with it:

I bow down before to the whole rising all things.  
I bow down before to the whole where all things are.  
I bow down before to the whole where all things get back.  
I bow down before to the inexpressible whole that always is.

---

<sup>3</sup> The concept of existence of qualitatively different evolutionary stages in the development of the separate branches of sciences, sciences and system of sciences is presented in Appendix 1. Appendix 2 contains an conceptional program for further investigations related as to the problem of geometrization of chemistry, as well as to the far more general problems concerning the essence and purpose of the scientific knowledge and the stages of development through which it passes. The Epilogue contains the referee’s report of Prof. Dr. Danail Bonchev and Prof. Dr. Ovanes Mekenyan and the responses of their remarks.

## FIGURES

**Fig. 1. Simplified short form of the Periodic System, denoted as starting table  $T_0$ .**

This table represents only the most general characteristics of the Periodic law (*i.e.* the ordering of chemical elements in groups and periods, as well as the ordering of the groups and the periods), but does not represent the atoms of chemical elements belonging to the corresponding A or B subgroups.

The approximate model  $P=f(S,T_0)$  originates from this table.

PERIODS	GROUPS (COLUMNS)										
	I	II	III	IV	V	VI	VII	VIII			
1	1 <b>H</b>							2 <b>He</b>	1		
2	3 <b>Li</b>	4 <b>Be</b>	5 <b>B</b>	6 <b>C</b>	7 <b>N</b>	8 <b>O</b>	9 <b>F</b>	10 <b>Ne</b>	2		
3	11 <b>Na</b>	12 <b>Mg</b>	13 <b>Al</b>	14 <b>Si</b>	15 <b>P</b>	16 <b>S</b>	17 <b>Cl</b>	18 <b>Ar</b>	3		
4	19 <b>K</b>	20 <b>Ca</b>	21 <b>Sc</b>	22 <b>Ti</b>	23 <b>V</b>	24 <b>Cr</b>	25 <b>Mn</b>	26 <b>Fe</b>	4		
								27 <b>Co</b>	5		
								28 <b>Ni</b>	6	<b>R</b>	
	29 <b>Cu</b>	30 <b>Zn</b>	31 <b>Ga</b>	32 <b>Ge</b>	33 <b>As</b>	34 <b>Se</b>	35 <b>Br</b>	36 <b>Kr</b>	7		
5	37 <b>Rb</b>	38 <b>Sr</b>	39 <b>Y</b>	40 <b>Zr</b>	41 <b>Nb</b>	42 <b>Mo</b>	43 <b>Tc</b>	44 <b>Ru</b>	8		
								45 <b>Rh</b>	9	<b>O</b>	
								46 <b>Pd</b>	10		
	47 <b>Ag</b>	48 <b>Cd</b>	49 <b>In</b>	50 <b>Sn</b>	51 <b>Sb</b>	52 <b>Te</b>	53 <b>I</b>	54 <b>Xe</b>	11		
6	55 <b>Cs</b>	56 <b>Ba</b>	57 <b>La</b>						12	<b>W</b>	
			lan- tha- ni- des						↓		
			71 <b>Lu</b>	72 <b>Hf</b>	73 <b>Ta</b>	74 <b>W</b>	75 <b>Re</b>	76 <b>Os</b>	26	<b>S</b>	
							77 <b>Ir</b>	27			
							78 <b>Pt</b>	28			
	79 <b>Au</b>	80 <b>Hg</b>	81 <b>Tl</b>	82 <b>Pb</b>	83 <b>Bi</b>	84 <b>Po</b>	85 <b>At</b>	86 <b>Rn</b>	29		
7	87 <b>Fr</b>	88 <b>Ra</b>	89 <b>Ac</b>						30		
			ac- ti- ni- des						↓		
			103 <b>Lr</b>	104 <b>Ku</b>	105 <b>Ns</b>				44		

**Lanthanides**

<b>III</b>
57 <b>La</b>
58 <b>Ce</b>
59 <b>Pr</b>
60 <b>Nd</b>
61 <b>Pm</b>
62 <b>Sm</b>
63 <b>Eu</b>
64 <b>Gd</b>
65 <b>Tb</b>
66 <b>Dy</b>
67 <b>Ho</b>
68 <b>Er</b>
69 <b>Tm</b>
70 <b>Yb</b>
71 <b>Lu</b>

**Actinides**

<b>III</b>
89 <b>Ac</b>
90 <b>Th</b>
91 <b>Pa</b>
92 <b>U</b>
93 <b>Np</b>
94 <b>Pu</b>
95 <b>Am</b>
96 <b>Cm</b>
97 <b>Bk</b>
98 <b>Cf</b>
99 <b>Es</b>
100 <b>Fm</b>
101 <b>Md</b>
102 <b>No</b>
103 <b>Lr</b>

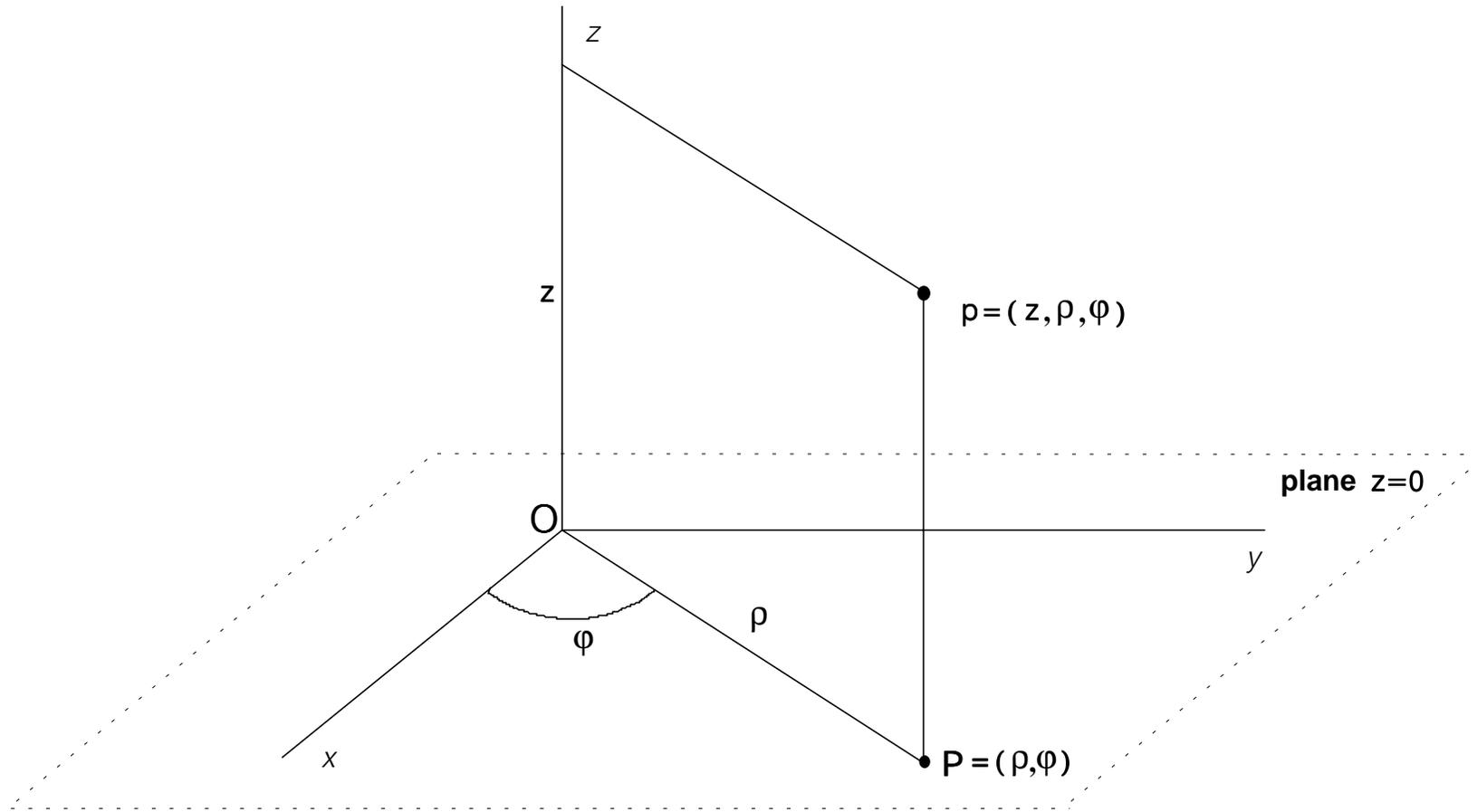
**Fig. 2.1. Long form of the Periodic System, denoted as starting table T<sub>1</sub>.**

In this table is presented the successive order of filling up of the electron shells of atoms as a function of the atomic number. The asterisks denote: \* - electron orbitals ns<sup>1</sup> and ns<sup>2</sup>; \*\* - electron orbitals np<sup>1</sup> and np<sup>2</sup>; \*\*\* - electron orbitals (n-1)d<sup>1</sup> → (n-1)d<sup>10</sup>; \*\*\*\* - electron orbitals (n-2)f<sup>0</sup>(n-1)d<sup>1</sup> → (n-2)f<sup>14</sup>. The more precise model Q=f<sub>1</sub>(S,T<sub>1</sub>) originates from this table.

		C O L U M N S																																					
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30	31	32						
P Å R Î D S	1	1 H	2 He																																	*	1	R O W S	
	2	3 Li	4 Be																																		*		2
	3			5 B	6 C	7 N	8 O	9 F	10 Ne																												**		3
	4	11 Na	12 Mg																																		*		4
	5			13 Al	14 Si	15 P	16 S	17 Cl	18 Ar																												**		5
	6	19 K	20 Ca																																	*	6		
	7										21 Sc	22 Ti	23 V	24 Cr	25 Mn	26 Fe	27 Co	28 Ni	29 Cu	30 Zn																	***		7
	8			31 Ga	32 Ge	33 As	34 Se	35 Br	36 Kr																												**		8
	9	37 Rb	38 Sr																																		*		9
	10										39 Y	40 Zr	41 Nb	42 Mo	43 Tc	44 Ru	45 Rh	46 Pd	47 Ag	48 Cd																	***		10
	11			49 In	50 Sn	51 Sb	52 Te	53 I	54 Xe																												**		11
	12	55 Cs	56 Ba																																		*		12
	13																					57 La	58 Ce	59 Pr	60 Nd	61 Pm	62 Sm	63 Eu	64 Gd	65 Tb	66 Dy	67 Ho	68 Er	69 Tm	70 Yb	****	13		
	14										71 Lu	72 Hf	73 Ta	74 W	75 Re	76 Os	77 Ir	78 Pt	79 Au	80 Hg																	***		14
	15			81 Tl	82 Pb	83 Bi	84 Po	85 At	86 Rn																												**		15
	16	87 Fr	88 Ra																																		*		16
	17																					89 Ac	90 Th	91 Pa	92 U	93 Np	94 Pu	95 Am	96 Cm	97 Bk	98 Cf	99 Es	100 Fm	101 Md	102 No	****	17		
	18										103 Lr	104 Ku	105 Ns																								***		18
type of elements	s		p						d										f																				

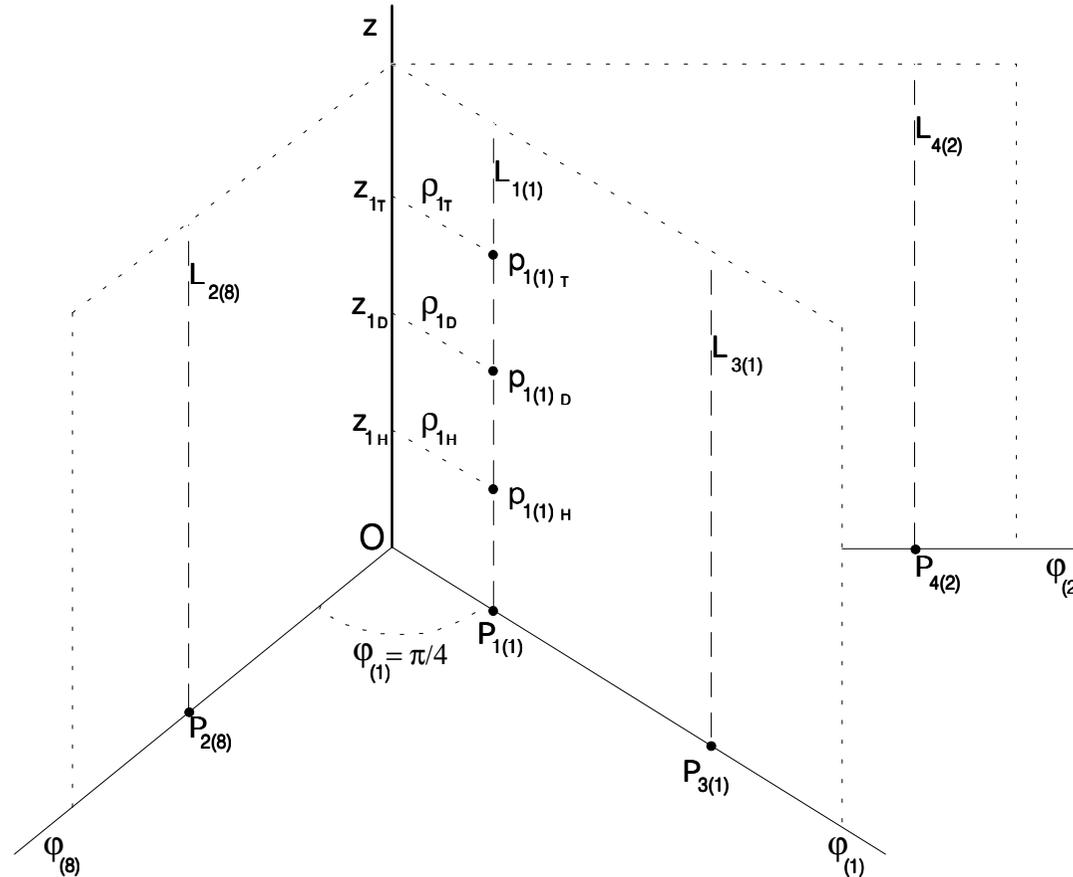


**Fig. 3. Cylindrical coordinates of the points  $p=(z,\rho,\varphi)$  with respect to Mendelev's coordinate system  $K_M$  in the Mendelev's space  $V_M(3)$ .**



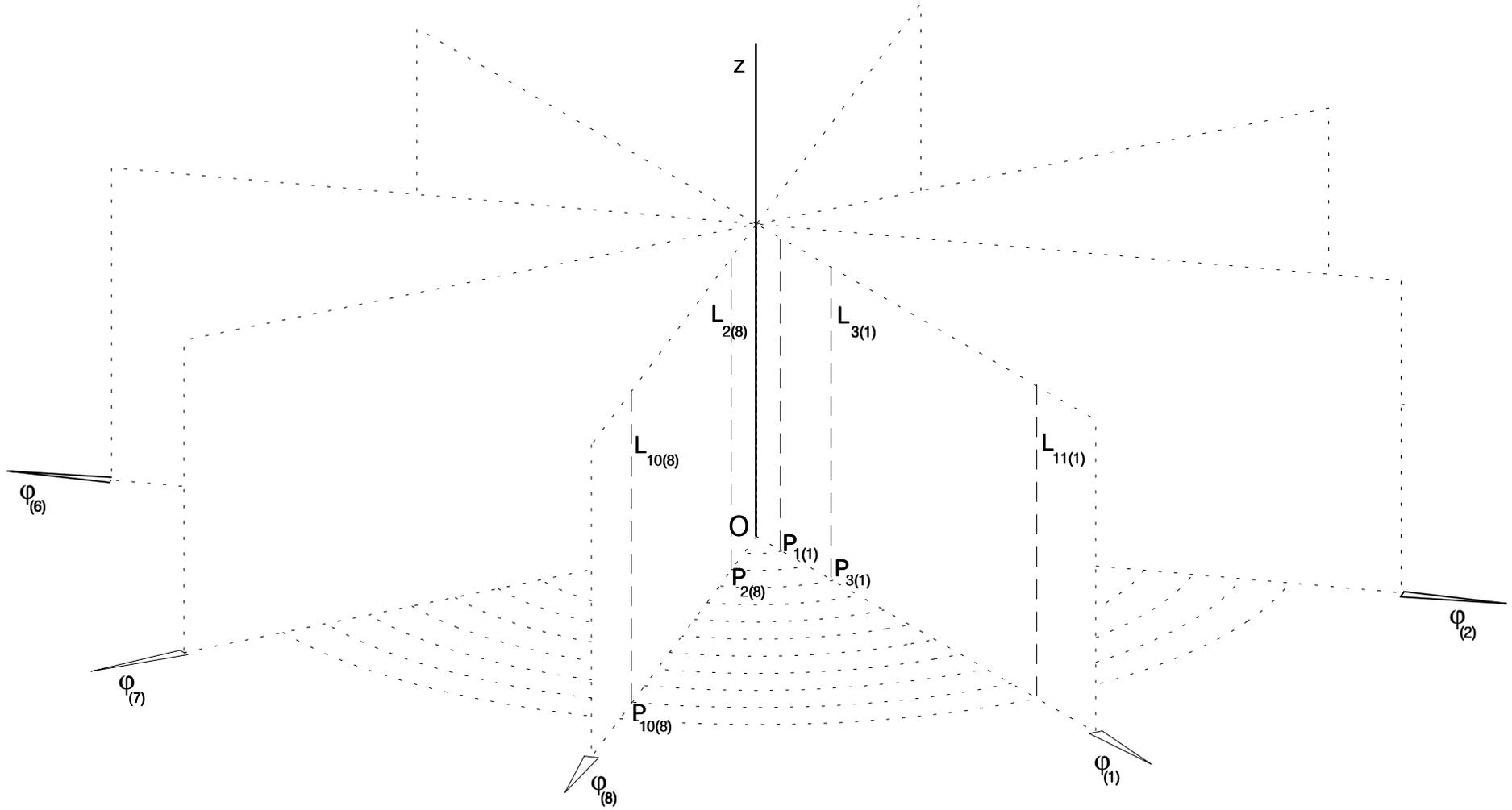
**Fig. 4. Mathematical representation of different species of atoms (isotopes) of the chemical element with atomic number  $i=1$ .**

Shown are: 1) Mendeleev's coordinate system  $K_M=Oz\rho\varphi$ ; 2) a parts of the semiplanes  $zO\varphi_{(1)}$ ,  $zO\varphi_{(2)}$  and  $zO\varphi_{(3)}$ ; 3) the points  $p_{1(1)H}$ ,  $p_{1(1)D}$  and  $p_{1(1)T}$ , *i.e.*, the three-dimensional images of the atoms of hydrogen (H), deuterium (D), and tritium (T); 4) the points  $P_{1(1)}$ ,  $P_{2(8)}$ ,  $P_{3(1)}$ , and  $P_{4(2)}$ , *i.e.* the two-dimensional images in the plane  $z=0$  of the elements with atomic numbers  $i=1,2,3,4$ ; 5) some of the lines  $L_{1(1)}$ ,  $L_{2(8)}$ ,  $L_{3(1)}$  and  $L_{4(2)}$ .



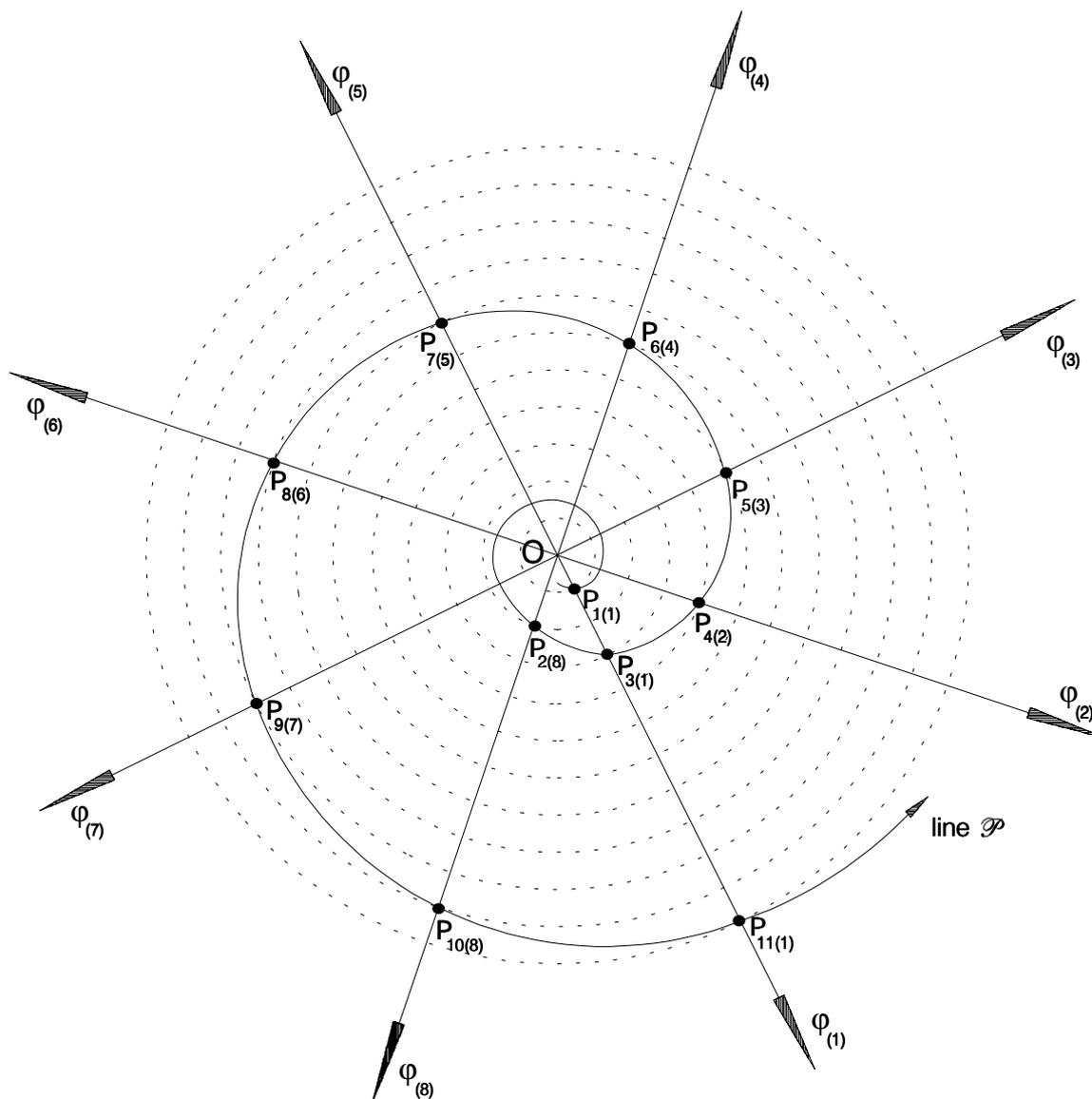
**Fig. 5. The semiplanes  $zO\varphi_{(j)}$  ( $j=1,2,\dots,8$ ) in  $V_M(3)$ .**

The dashed lines in the plane  $z=0$  show circumferences of radii  $\rho_i=1,2,\dots,11$ , which contain the two-dimensional images of the first eleven chemical elements, i.e. the points  $P_{i(j)}$  ( $i=1,2,\dots,11$  and  $j=1,2,\dots,8$ ).



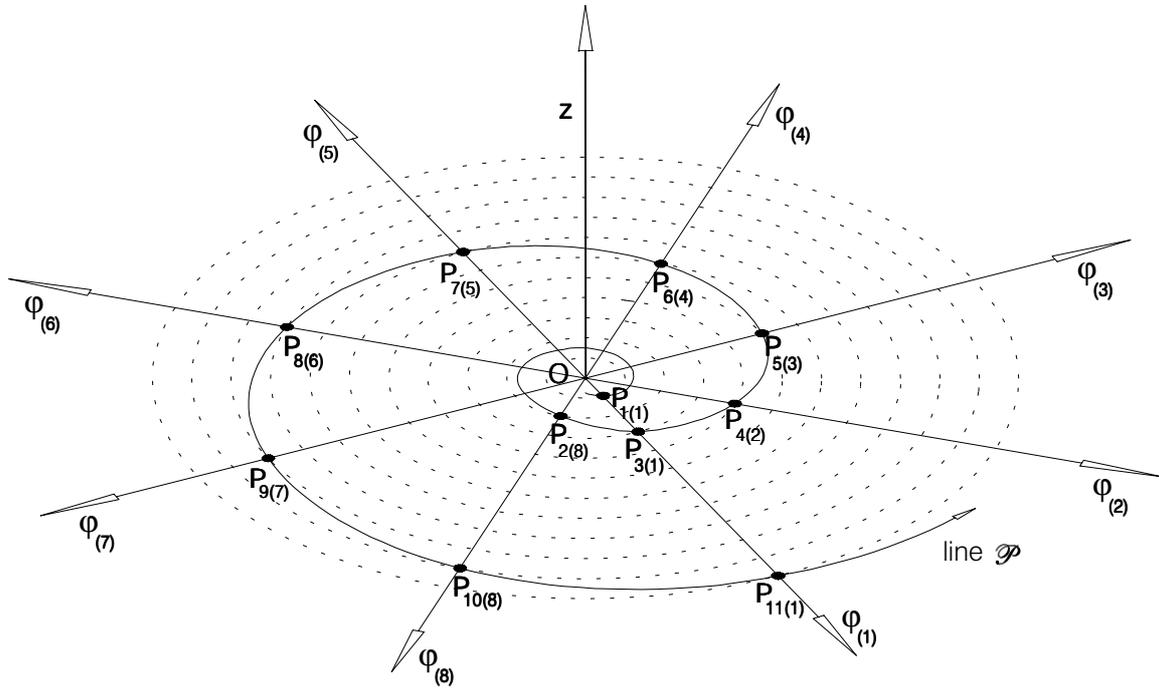
**Fig. 6.1. Construction of the auxiliary line  $\mathcal{P}$  in the plane  $z=0$  of  $V_M(3)$ .**

Shown is the starting part of the line  $\mathcal{P}$ , where it passes through the points  $P_{i(j)}$  of numbers  $i=1,2,\dots,11$  and  $j=1,2,\dots,8$  (i.e., through the two-dimensional images of the first 11 chemical elements). The radii  $\rho_i$  of the dashed circumferences are equal to  $1,2,\dots,11$ , respectively.

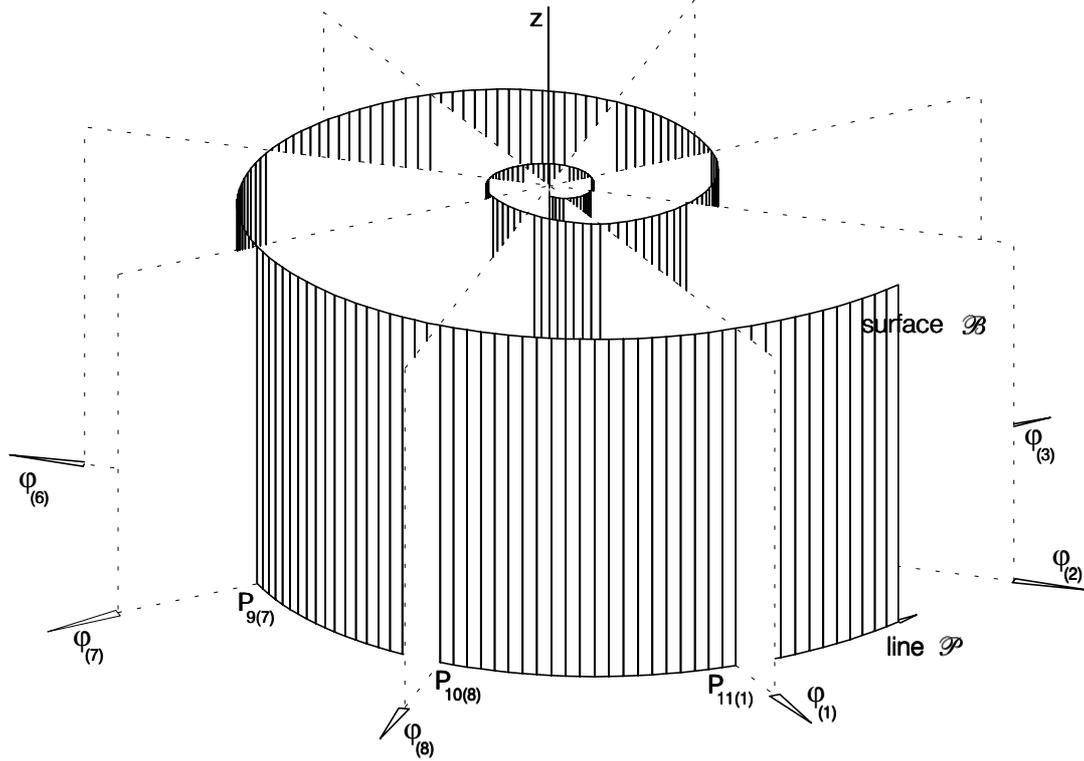


**Fig. 6.2. Construction of the auxiliary line  $\mathcal{P}$  in the plane  $z=0$  of  $V_M(3)$ .**

View point is outside the axis  $Oz$ .

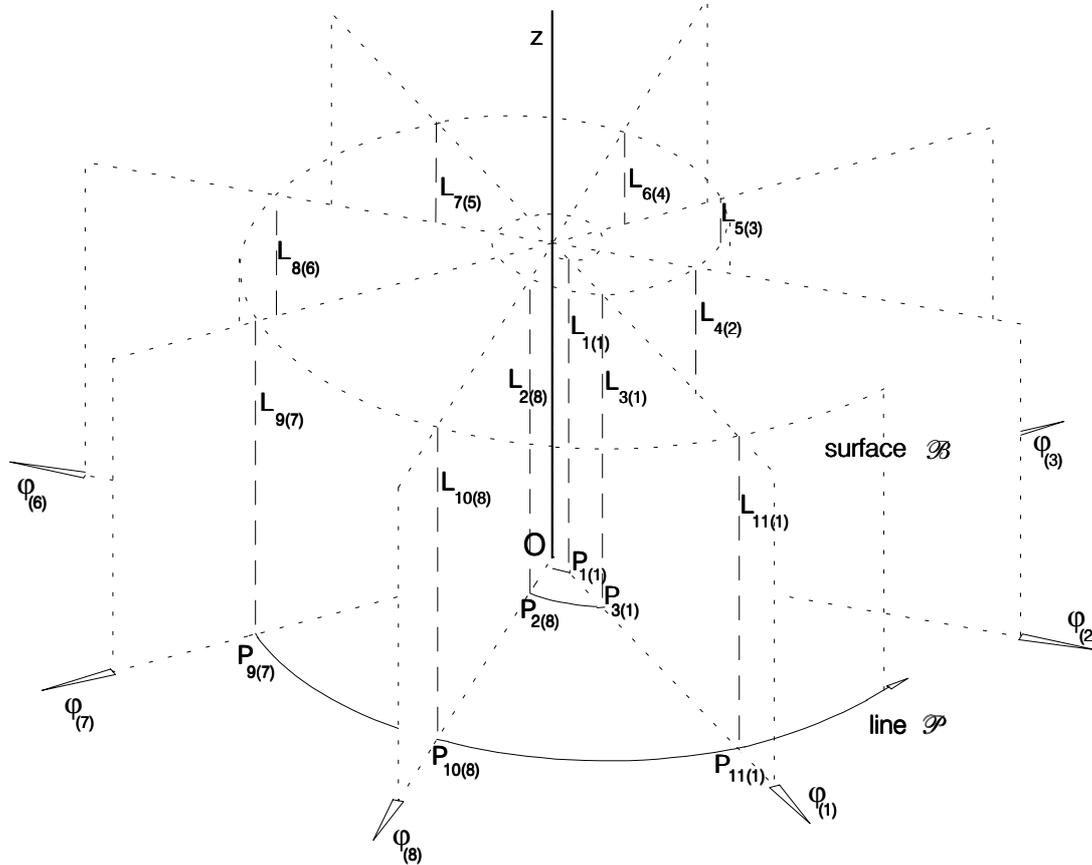


**Fig. 7.1. Construction of the auxiliary surface  $\mathcal{B}$  in  $V_M(3)$ .**

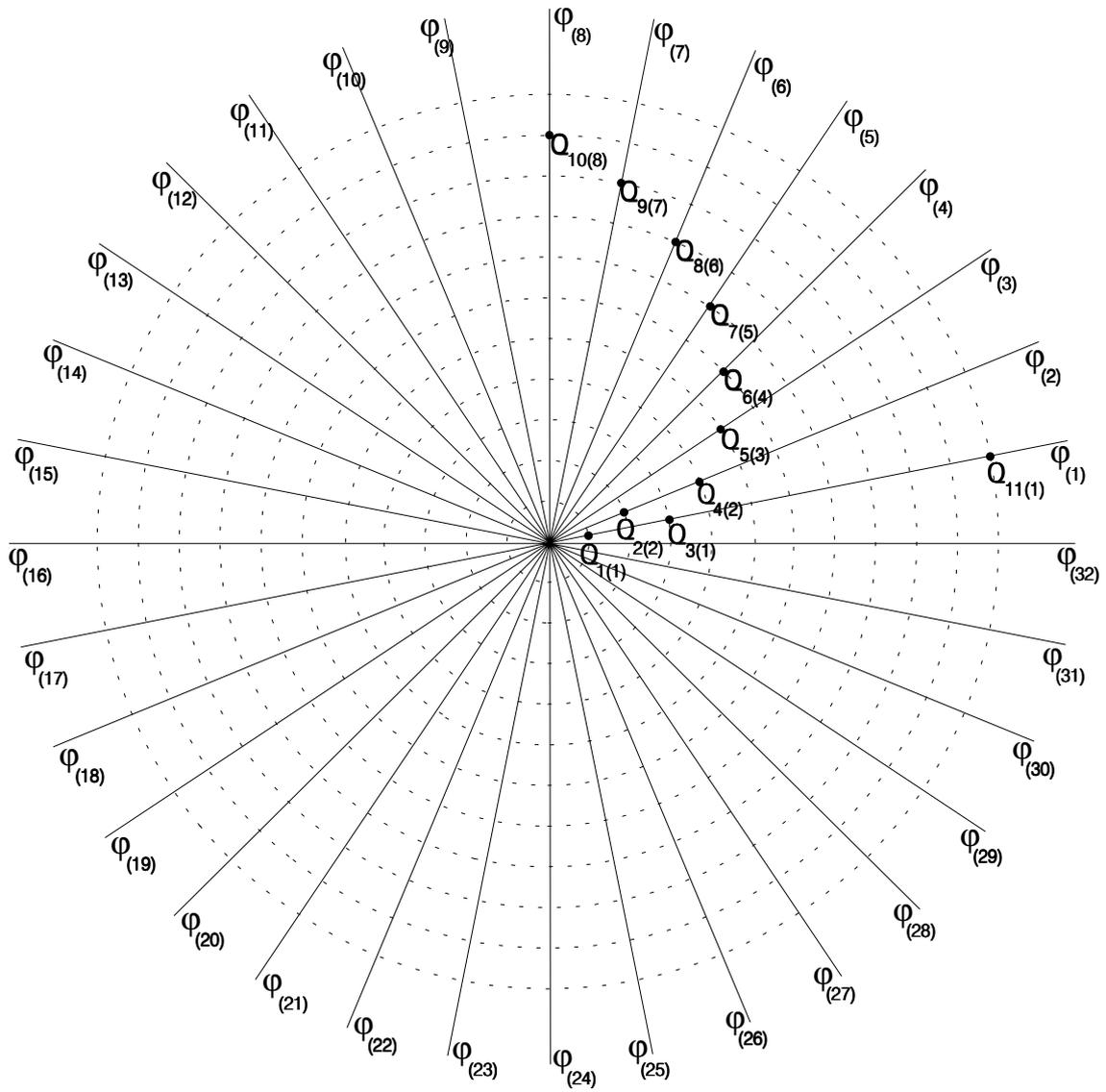


**Fig. 7.2. Auxiliary surface  $\mathcal{B}$  in  $V_M(3)$ .**

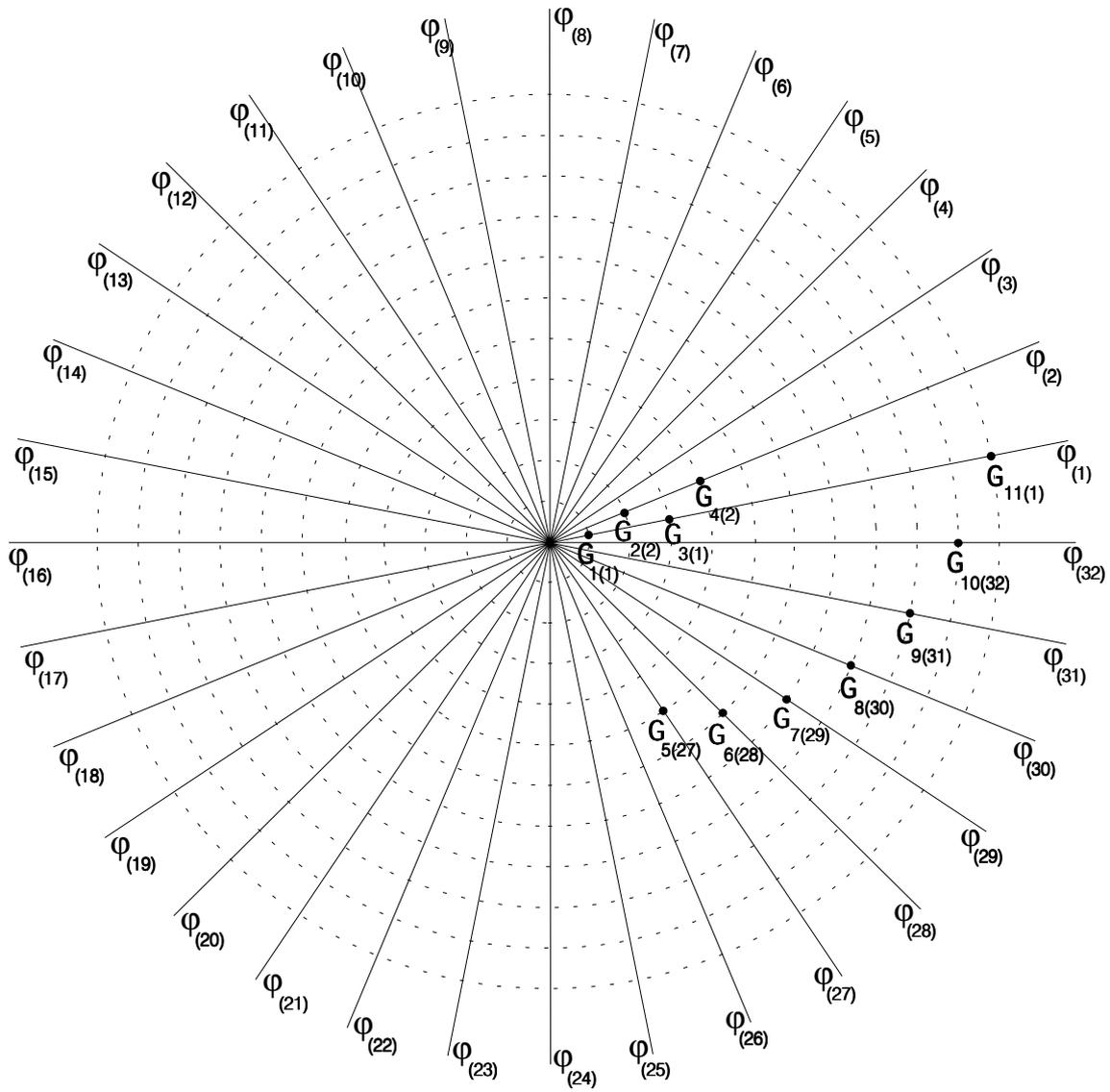
Shown is the starting part of the surface  $\mathcal{B}$ , where it passes through the lines  $L_{i(j)}$  of numbers  $i=1,2,\dots,11$  and  $j=1,2,\dots,8$ .



**Fig. 8.1. Two-dimensional mathematical images in the plane  $z=0$  of the first 11 chemical elements in the more precise mathematical model  $Q=f_1(S,T_1)$ .**

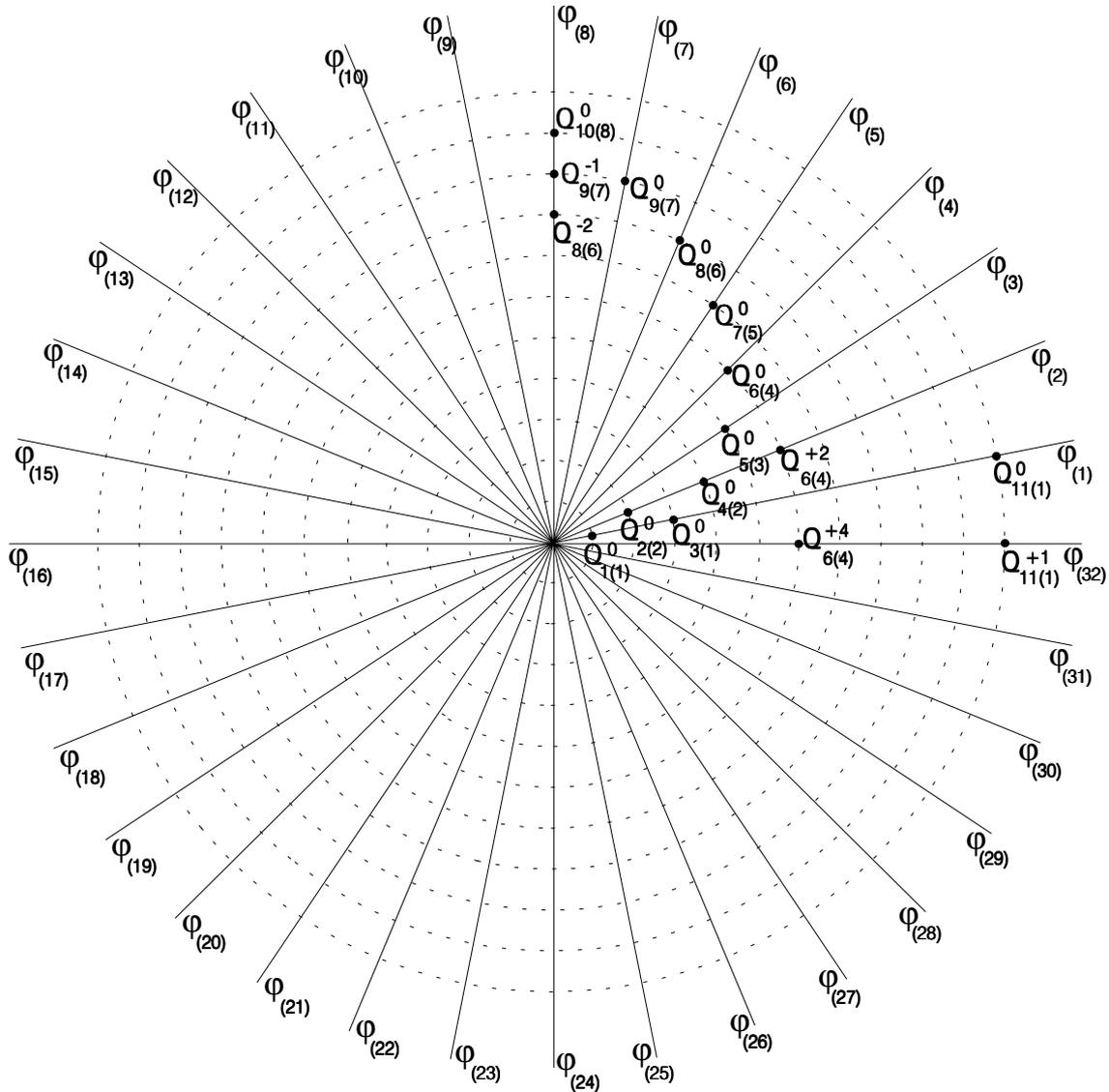


**Fig. 8.2. Two-dimensional mathematical images in the plane  $z=0$  of the first 11 chemical elements in the more precise mathematical model  $G=f_1(S,T_2)$ .**



**Fig. 9.1. Two-dimensional mathematical images in the plane  $z=0$  of atoms and monatomic ions in the generalized mathematical model  $Q = F_i(S, T_i)$ .**

Shown are: 1) two-dimensional mathematical images  $Q_{i(j)}^h$  of the atoms ( $h=0$ ) of first 11 chemical elements ( $i=1, \dots, 11$ ); 2) two-dimensional mathematical images  $Q_{6(4)}^{+2}$ ,  $Q_{6(4)}^{+4}$ ,  $Q_{8(6)}^{-2}$ ,  $Q_{9(7)}^{-1}$  and  $Q_{11(1)}^{+1}$  of the following species of monatomic ions  $C^{+2}$ ,  $C^{+4}$ ,  $O^{-2}$ ,  $F^{-1}$  and  $Na^{+1}$ .



**Fig. 9.2. Two-dimensional mathematical images in the plane  $z=0$  of atoms and monatomic ions in the generalized mathematical model  $G = F_1(S, T_2)$ .**

Shown are: 1) two-dimensional mathematical images  $G_{i(j)}^h$  of the atoms ( $h=0$ ) of first 11 chemical elements ( $i=1, \dots, 11$ ); 2) two-dimensional mathematical images  $G_{6(28)}^{+2}$ ,  $G_{6(28)}^{+4}$ ,  $G_{8(30)}^{-2}$ ,  $G_{9(31)}^{-1}$  and  $G_{11(1)}^{+1}$  of the following species of monatomic ions  $C^{+2}$ ,  $C^{+4}$ ,  $O^{-2}$ ,  $F^{-1}$  and  $Na^{+1}$ .

